Management Production and Inventories: From EOQ/EPQ to ELSP and its extension with shelf life and transitive demand items

WP8 Agri-food supply chain decision-making under uncertainty

Teaching Session

Pilar I. Vidal-Carreras

Secondment in Riviera, July-August 2019

Introduction

- At this session, the economic order quantity (EOQ) as one of the simplest and used models to control production and inventory is presented. After that, is presented Economic Lot Scheduling Problem which occurs when EPQ deals with more than one item. Some heuristics to implement ELSP approach are described. After that the particular case of a firm which package fresh vegetables is presented, describing the special characteristics of this framework regarding the management of the production and inventories.

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Sections

- EOQ origins and framework
- EOQ vs EPQ
- EOP/EPQ to production and inventory management policies
- ELSP and solving methods
- Extensions and its implications on a tier 2 (agri-food supply chain)

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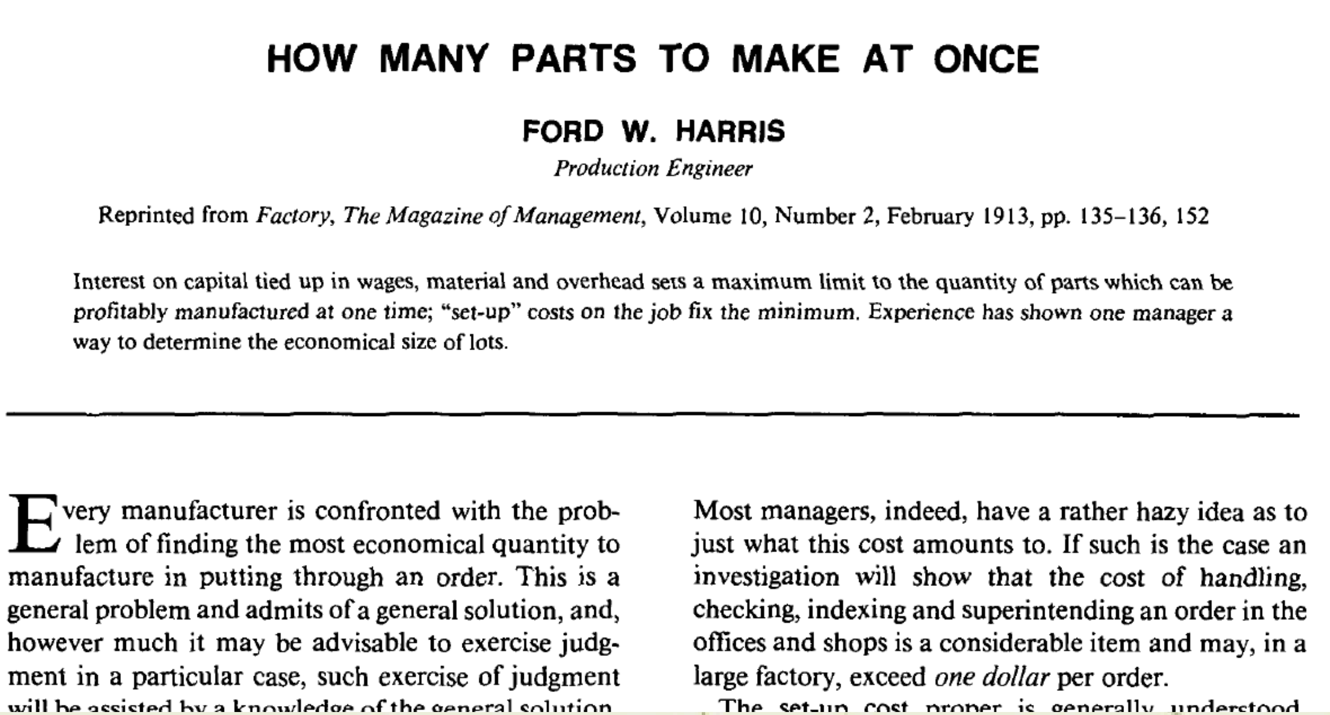
EOQ origins

- In February 1913, Harris published in the "*Factory, The Magazine of Management" a paper which open a wide research area*.*
- *Paper starts:*



HOW MANY PARTS TO MAKE AT ONCE

FORD W. HARRIS
Production Engineer

Reprinted from *Factory, The Magazine of Management*, Volume 10, Number 2, February 1913, pp. 135–136, 152

Interest on capital tied up in wages, material and overhead sets a maximum limit to the quantity of parts which can be profitably manufactured at one time; "set-up" costs on the job fix the minimum. Experience has shown one manager a way to determine the economical size of lots.

Every manufacturer is confronted with the problem of finding the most economical quantity to manufacture in putting through an order. This is a general problem and admits of a general solution, and, however much it may be advisable to exercise judgment in a particular case, such exercise of judgment will be assisted by a knowledge of the general solution.

Most managers, indeed, have a rather hazy idea as to just what this cost amounts to. If such is the case an investigation will show that the cost of handling, checking, indexing and superintending an order in the offices and shops is a considerable item and may, in a large factory, exceed *one dollar* per order.

The set-up cost proper is generally understood

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EOQ framework

- Production or Inventory systems in which exists:
  - A demand rate for product (known and constant)
  - A cost (and a time) involved in releasing an order (administration, handling, setup..) → decrease with large lots
  - A cost to storage the order → decrease with small lot
  - An unit cost for acquire or produce an item

EOQ searches the balance between order/setup cost and holding inventory cost considering also cost for adquisition or manufacturing.

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EOQ framework

- Assumptions:
  - A system which order or produce an item.
  - The demand rate D for the product is known and constant.
  - Lead/Setup times (ts) and setup costs (A) are fixed and known.
  - The unit cost for the product (c) is known.
  - Holding costs h are considered proportional to the units stored

EOQ searches the balance between order/setup cost and holding inventory cost considering also cost for adquisition or manufacturing.

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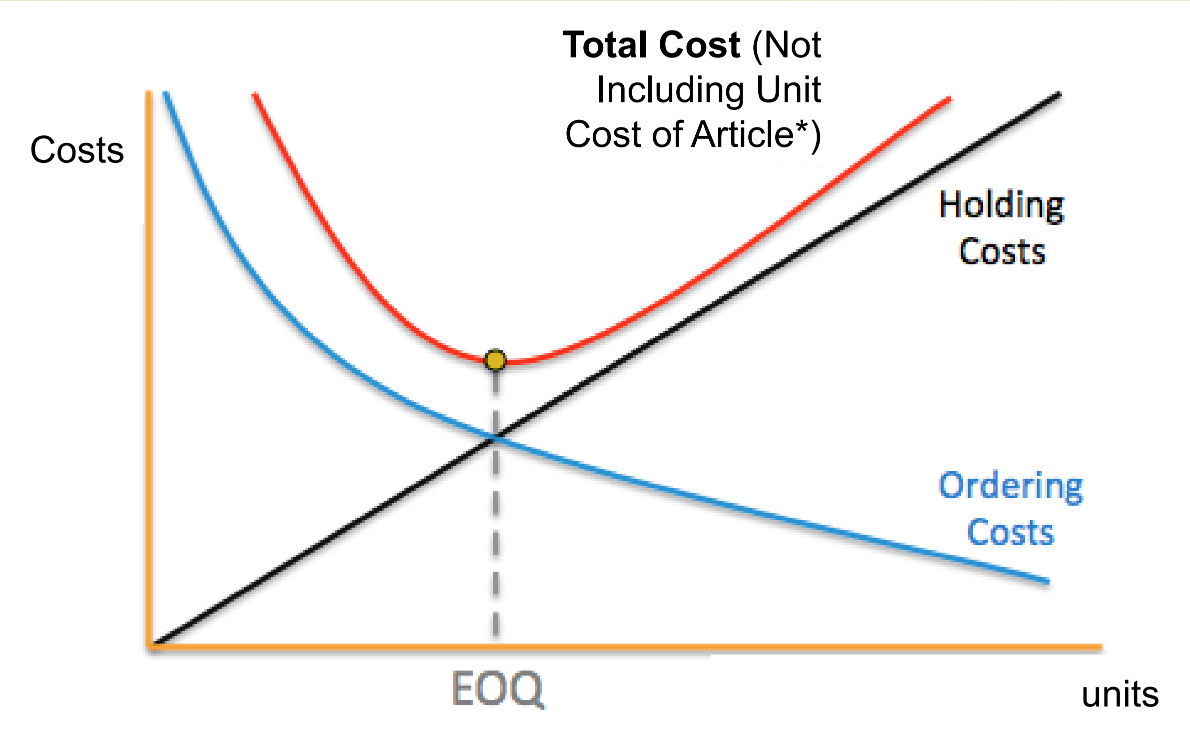
EOQ formula



$$TC(Q) = \frac{D}{Q}A + \frac{Q}{2}\text{h}$$

D = annual demand for the item
A = ordering costs for an order
h = holding cost per unit per year
TC = annual cost of ordering+ annual cost of holding inventory

$$\frac{dTC}{dQ} = -\frac{D}{Q^2} + \frac{h}{2} \rightarrow \min \frac{dTC}{dQ} = 0$$

$$Q = \sqrt{\frac{2AD}{h}} \Rightarrow \text{EOQ}$$

* Usually is not considered because is independent of Q and their derivate is 0, so not affect EOQ formula

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EOQ/EPQ

Economic Order Quantity (EOQ):

- Harris (1913)
- Used when products are obtained from an outside supplier

  → Assumption Instantaneous replenishment

$$Q = \sqrt{\frac{2AD}{H}}$$

Economic Production Quantity (EPQ):

- Taft (1918)
- Used when products are internally manufactured instead of being obtained from an outside supplier

  → Assumption that the order is received at a constant finite rate over time.

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EPQ framework

- Assumptions:
  - A system which order or produce an item.
  - The demand rate D for the product is known and constant.
  - Lead/Setup times (ts) and setup costs (A) are fixed and known.
  - The unit cost for the product (c) is known.
  - Holding costs h are considered proportional to the units stored
  - The production rate (p) for the product is known and it is higher than its demand rate

EOQ searches the balance between setup cost and holding inventory cost considering also cost for adquisition or manufacturing.

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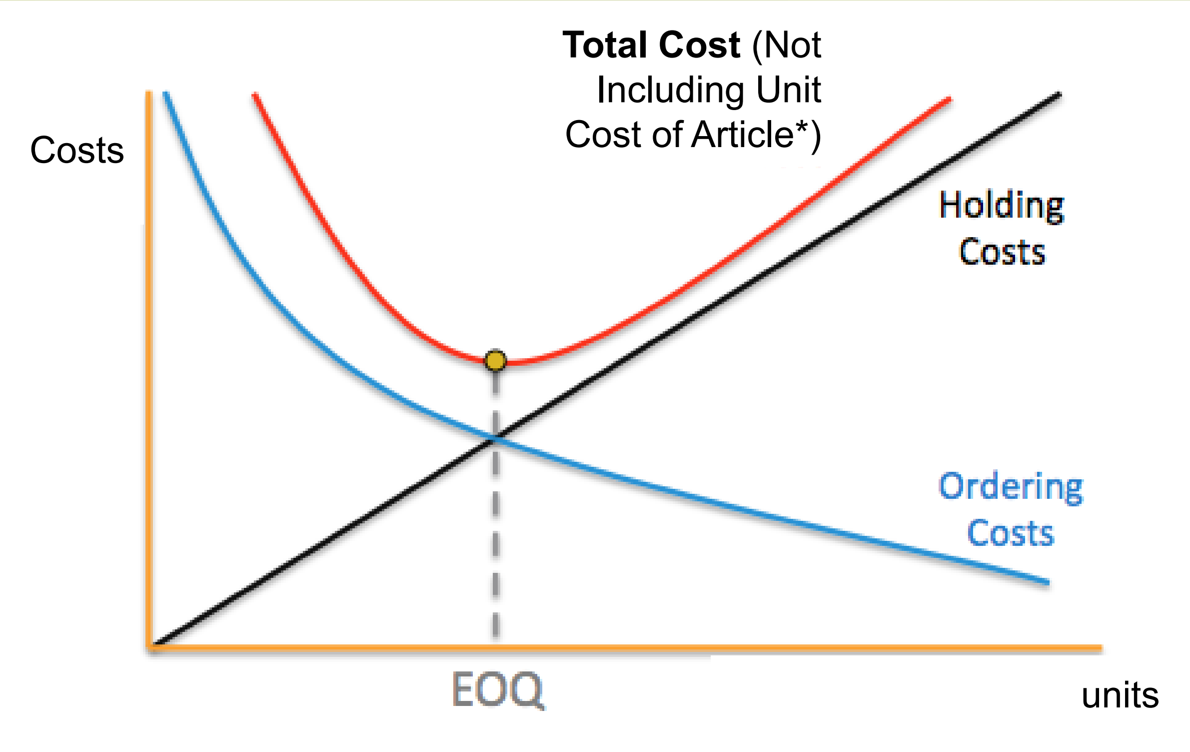
EPQ formula



$$TC(Q) = \frac{D}{Q}A + \frac{Q\left(1 - \frac{d}{p}\right)}{2}h$$

D = annual demand for the item
A = ordering costs for an order
h = holding cost per unit per year
p=production rate
TC = annual cost of ordering+ annual cost of holding inventory

$$\frac{dTC}{dQ} = -\frac{D}{Q^2} + \frac{\left(1-\frac{d}{p}\right)h}{2} \rightarrow \min \frac{dTC}{dQ} = 0$$

$$Q = \sqrt{\frac{2AD}{h\left(1 - \frac{d}{p}\right)}}$$ → EPQ

* Usually is not considered because is independent of Q and their derivate is 0, so not affect EOQ formula

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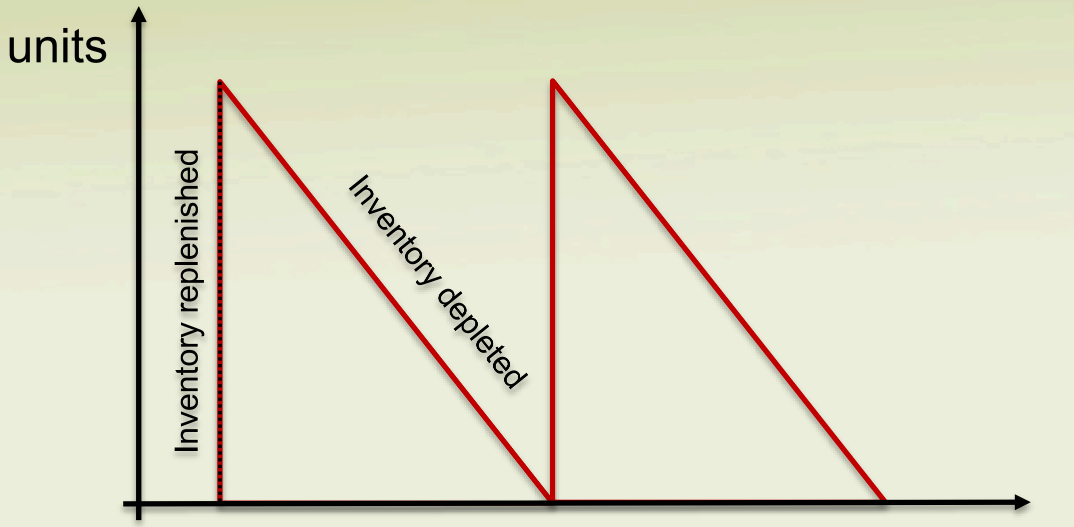
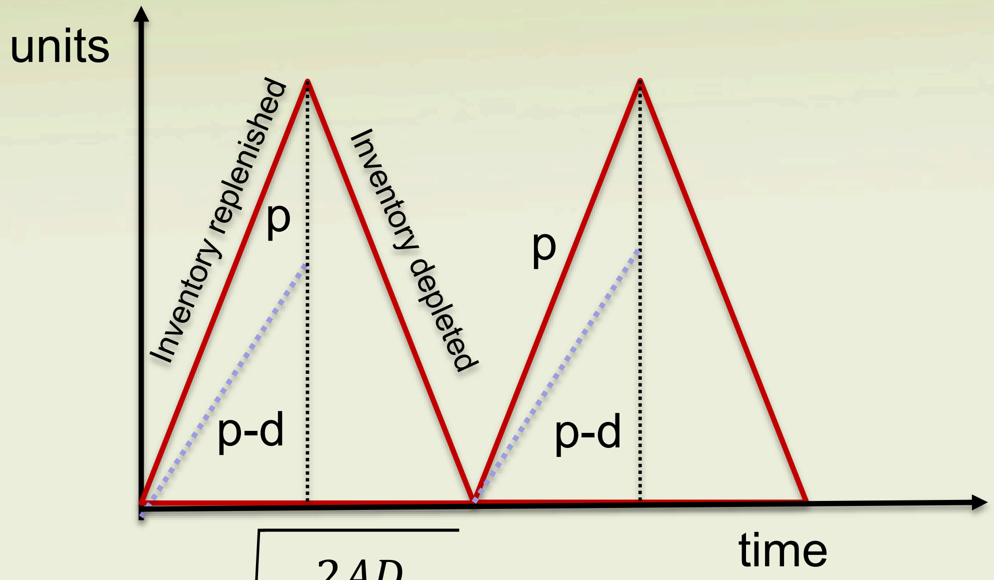
EPQ / EOQ

d = demand for the item per unit time
p = production rate for the item per unit time



$$Q = \sqrt{\frac{2AD}{H}}$$



$$Q = \sqrt{\frac{2AD}{h\left(1 - \frac{d}{p}\right)}}$$

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From EPQ to a Production and Inventory Managament Policy

EOQ /EPQ

Production and Inventory Managament Policy

$$Q = \sqrt{\frac{2AD}{H}}$$

$$Q = \sqrt{\frac{2AD}{h\left(1 - \frac{d}{p}\right)}}$$

- When produce/order?
- How much produce/order?

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From EPQ to a Production and Inventory Managament Policy

Questions determine by the production and inventory management policy:

- When produce/order?
- How much produce/order?

From EOQ to a Production and Inventory Managament Policy

Questions determine by the production and inventory management policy:

- When produce/order?
  - setting a minimium stock level **s** at which to reorder
  - stablishing specific moments of time when to reorder (**T**)

- how much produce/order?

s,T calculated using EOQ/EPQ

Cicle Time → $Q = DT$

From EOQ to a Production and Inventory Managament Policy

Questions determine by the production and inventory management policy:

- When produce/order?
- how much produce/order?
  - considering a stock level **S** that serves as a reference to fill
  - fixed quantity **Q** that optimizes the affected costs

S calculated using EOQ/EPQ and Q is EOQ/EPQ

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From EOQ/EPQ to a Production and Inventory Managament Policy

EOQ /EPQ

Production and Inventory Managament Policy

$$Q = \sqrt{\frac{2AD}{H}}$$

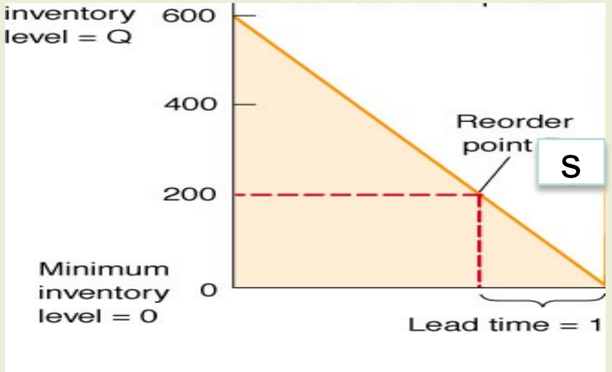
$$Q = \sqrt{\frac{2AD}{h\left(1 - \frac{d}{p}\right)}}$$

Reorder point (s,Q): order/produce Q when inventory level arrives s

Model (s,S): order/produce to S when inventory level arrives s

Periodic review (T,S): order/produce every T to S

Model (T,Q): order/produce every T the quantity Q



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Deterministic / Stochastic Demand

- Assumptions:
  - A system which order or produce an item.
  - The demand rate D for the product is known and constant.
  - Lead/Setup times (ts) and setup costs (A) are fixed and known.
  - The unit cost for the product (c) is known.
  - Holding costs (h) are considered proportional to the units stored
  - The production rate (p) for the product is known and it is higher than its demand rate.

Deterministic / Stochastic Demand

- Assumptions:
  - A system which order or produce an item.
  - The demand rate for the product is stochastic stationary with mean d
  - Lead/Setup times (ts) and setup costs (A) are fixed and known.
  - The unit cost for the product (c) is known.
  - Holding costs (h) are considered proportional to the units stored
  - The production rate (p) for the product is known and it is higher than its demand rate.

Safety Stocks **ss** in policies

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Multi-item EPQ: ELSP

- When more than one item has to be produced in a machine with finite capacity, starts a “fight” between items for the capacity of the machine, and a real sequencing problem starts.
- FEASIBILITY PROBLEM

Economic **L**ot **S**cheduling **P**roblem

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ELSP

- The ELSP *(Rogers, 1958; Bomberger, 1966; Madigan, 1968)* programming several items *i=1..g* on a single facility, where only one product can be produced at a time, with the objective of minimize the sum of holding costs and setup costs.
  - The production rate per item, p_i is deterministic and constant.
  - Production setup times A_i, and setup costs c_i, are independent of the production sequence.
  - Inventory holding costs h_i, are proportional to the inventory levels I_i.
  - Product demand rates d_i are deterministic or stochastic, but stationary – SELSP (stochastic lot scheduling problem)
  - Production capacity is sufficient to meet demand, and all the demand has to be served.

i,n	Item indexes i,n=1..g
d_i	Demand rate
p_i	Production rate
A_i	Setup Cost
h_i	Holding Cost
c_i	Setup time
T_i	Cycle length
I_i	Available inventory of item i
TC	Total Cost

NP-hard (Hsu, 1983)

SELSP (stochastic lot scheduling problem)

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ELSP

- Objective: a programme in which several items are produced on the same facility on a repetitive basis.
  - Lotsizing:
    - Different Approaches to determine Qi (order quantity) or Ti (cycle length) por each item
  - Sequencing Problem:
    - Traditional Models with multi-items: (si,Si), (si, Qi)..
    - RO-based (Segersted, 1999) heuristic to define the sequence
    - Other Heuristics

i,n	Item indexes i,n=1..g
d_i	Demand rate
p_i	Production rate
A_i	Setup Cost
h_i	Holding Cost
c_i	Setup time
T_i	Cycle length
I_i	Available inventory of item i
TC	Total Cost

NP-hard (Hsu, 1983)

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ELSP - Lotsizing

$$T_i = \sqrt{\frac{2A_i}{h_i d_i\left(1-\frac{d_i}{p_i}\right)}}$$

- EPQ, called Independent Solution – IS (Harris, 1913)
  - cycles of independent manufacturing quantities

→ LOWER BOUND

$Q_i = d_i T_i$

$$Q_i = \sqrt{\frac{2A_i d_i}{h_i\left(1-\frac{d_i}{p_i}\right)}}$$

- Common Cycle Approach (Hanssmann, 1962)
  - Same cycle time for each product, enough bigger to fit one production of every product
    - » if holding cost == setup cost

→ UPPER BOUND

→ Near optimal $Q_i = d_i T$

$$T = \sqrt{\frac{2\sum_{i=1}^{g} A_i}{\sum_{i=1}^{g} h_i d_i\left(1-\frac{d_i}{p_i}\right)}}$$

- Basic Period Approach (Bomberger, 1966)
  - Each product has its cycle time, but all the cycle time a multiple of a T period basic

→ heurisitical procedure for obtaining each Ti (Doll and Whybark, 1973)

$Q_i = d_i T_i^e$ $T_i^e = k_i T_{pb}$

- Time Varying Lot size (Maxwell, 1964)
  - Each product can be produced several times and with different batch sizes in a cyclic program

→ $Q_i = d_i T_i^v$ T_i

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ELSP – Scheduling: Basic Models

Reorder point (s,Q) with ss* :
order/produce Q when inventory level arrives s

$$s_i = ss_i + c_i d_i$$

Model (s,S) with ss* : order/produce to S when inventory level arrives s

$$S_i = s_i + T_i d_i \left(1 - d_i / p_i\right)$$

Periodic review (T,S) with ss* :
order/produce every T to S

$$Q_i = d_i T_i$$

Model (T,Q) with ss* : order/produce every T the quantity Q

i,n	Item indexes i,n=1..g
d_i	Demand rate
p_i	Production rate
A_i	Setup Cost
h_i	Holding Cost
c_i	Setup time
T_i	Cycle length
I_i	Available inventory of item i
TC	Total Cost

T,Q:
- EPQ
- Common Cycle Approach
- Basic Period Approach
- Time Varying Lot size

Run Out: refers to the demand time units that are available in inventory.

$$RO_i = \frac{I_i}{d_i} \qquad RO_i = \frac{I_i - ss_i}{d_i}$$

$$RO_i = \frac{I_i\left(1 - d_i / p_i\right)}{d_i} \qquad RO_i = \frac{I_i}{d_i} - c_i$$

$$RO_i < RO_{i'} < RO_{i''} < \ldots < RO_n \qquad i, i', i'' \in \{1 \ldots n\}$$

* If the demand is stochastic

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ELSP – Extensions

Variants about
all the assumptions
and more

The production rate per item, p_i is deterministic and constant.	Production Rate stochastic Production Rate flexible Two products at the same time, coproduction ….
Production setup times A_i, and setup costs c_i, are independent of the production sequence.	Dependents Setups Unkown costs …
Inventory holding costs h_i, are proportional to the inventory levels I_i.	Deteoriating holding cost, Time variable holding costs… Unkown costs Items to order
Product demand rates d_i are deterministic or stochastic, but stationary – SELSP (stochastic lot scheduling problem)	Demand stochastic stationary or not stationary …
Production capacity is sufficient to meet demand, and all the demand has to be served.	Shortages: Back orders (total/,partial), lost sales …

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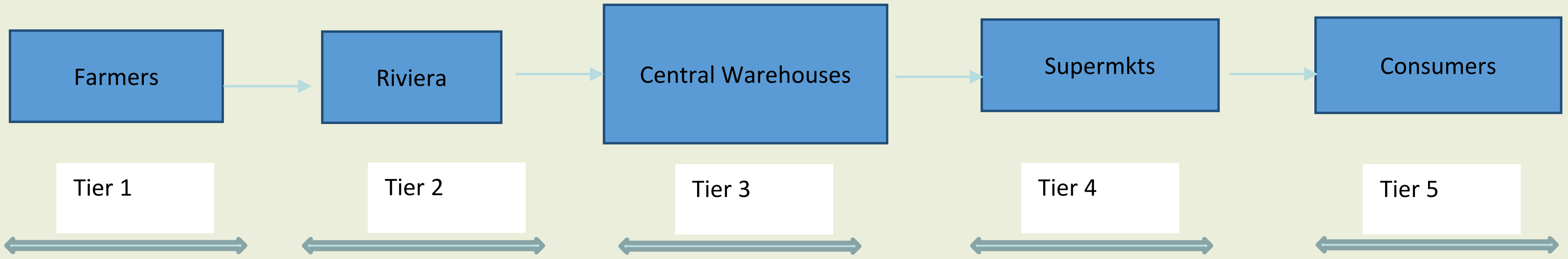
Extensions on ELSP accordind a fresh vegetable producer

- Uncertainity on raw material
- Shelf Life, deteriorating items or perishable ítems
- Mix of product with substitutability and transitive



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