Quantitative Models for Supply Chain Design and Management

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Profile

Raul Poler

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Professor at Polytechnic University of Valencia (UPV)

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Director of Master in Industrial Engineering and Logistics (MUIOL)

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Outline

- Introduction
- Mathematical Programming Models
- Strategic Models for Supply Chain Design
  - The single-source facility location problem
  - The distribution system problem
  - The integrated production/distribution system problem
- Tactical Models for Supply Chain Planning
  - The integrated production/distribution planning problem
  - The multisite supply chains planning problem
- Operational Models for Supply Chain Scheduling
  - The processing unit scheduling problem

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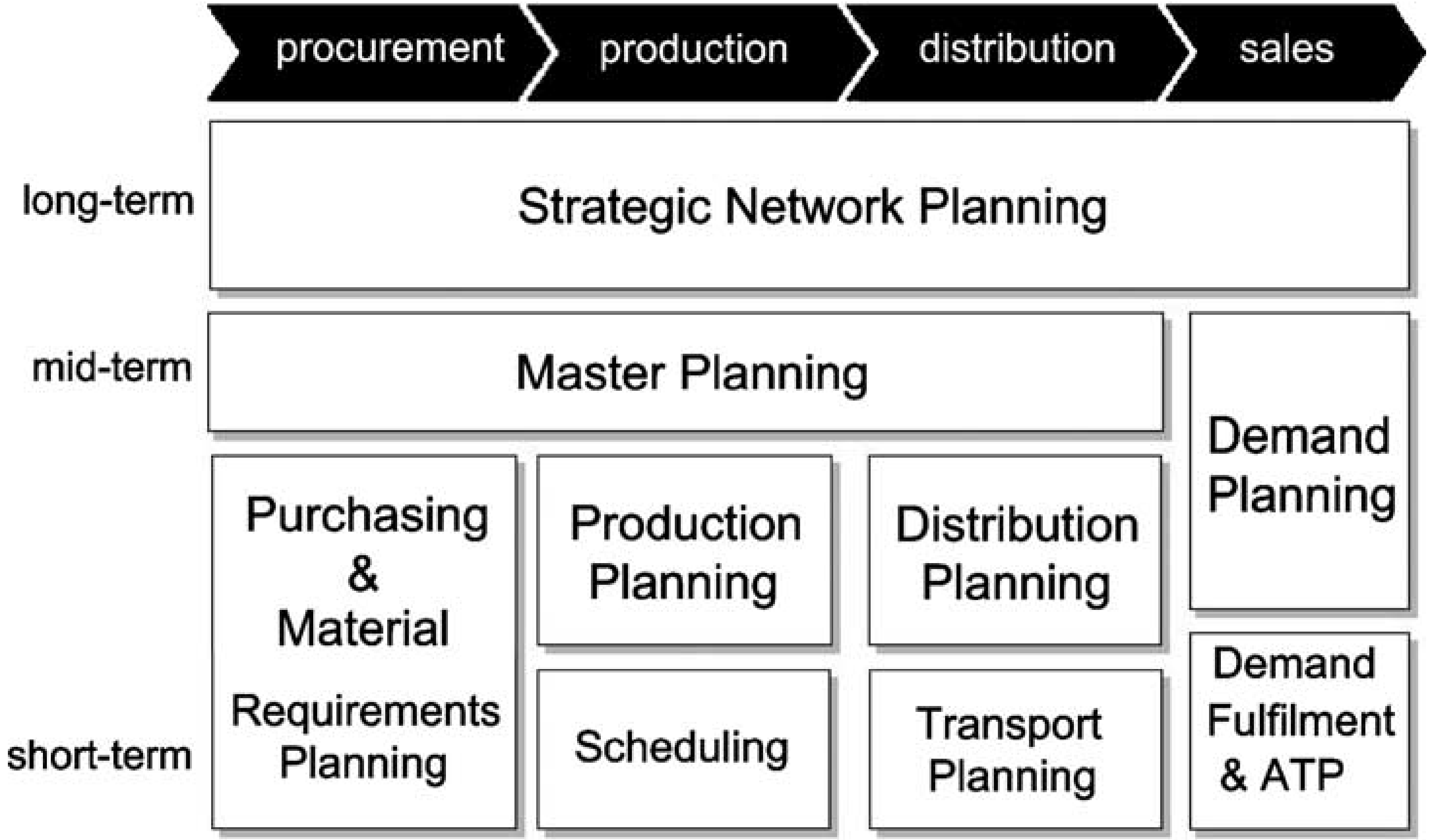
Introduction

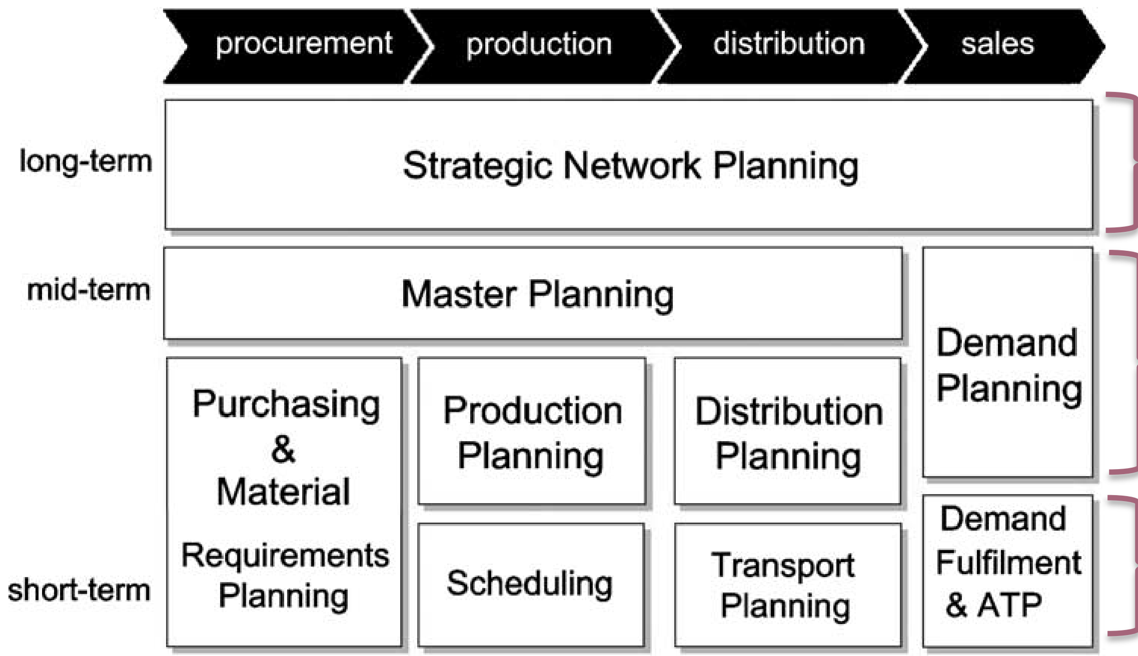
The Supply Chain planning matrix



Meyr, H., Wagner, M., Rohde, J., 2002. Structure of advanced planning systems. In: Stadtler, H., Kilger, C. (Eds.), Supply Chain Management and Advanced Planning–Concepts, Models Software and Case Studies, Berlin, pp. 99–104

Introduction

The Supply Chain planning matrix

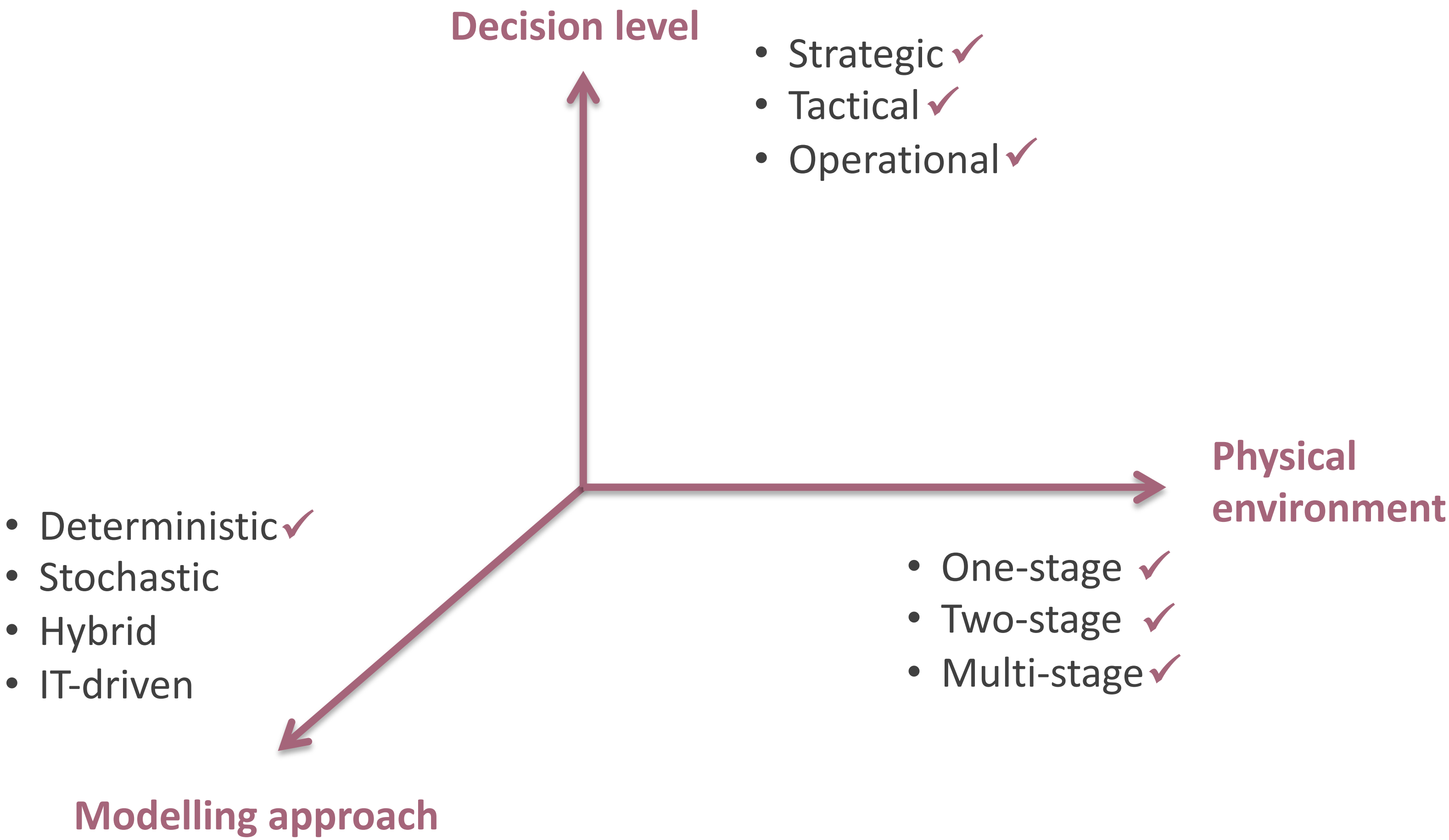


Quantitative Models

- Strategic Level
  - Supply Chain Design
- Tactical Level
  - SC Operations Planning
- Operational Level
  - SC Operations Scheduling

Introduction

Quantitative Models for SCM – Taxonomy:



Introduction

- Quantitative Models for SCM – Taxonomy:
  - Decision level
    - **Strategic:**
      - Horizon of 5-10 years
      - It affects long-term system performance
      - System Design
      - Resource Acquisition
    - **Tactical:**
      - Horizon 1-2 years
      - Medium term affects system performance
      - Deciding on the best use of different acquired resources
    - **Operational:**
      - 1-18 months horizon
      - Very small time periods
      - Sequencing and timing of operations

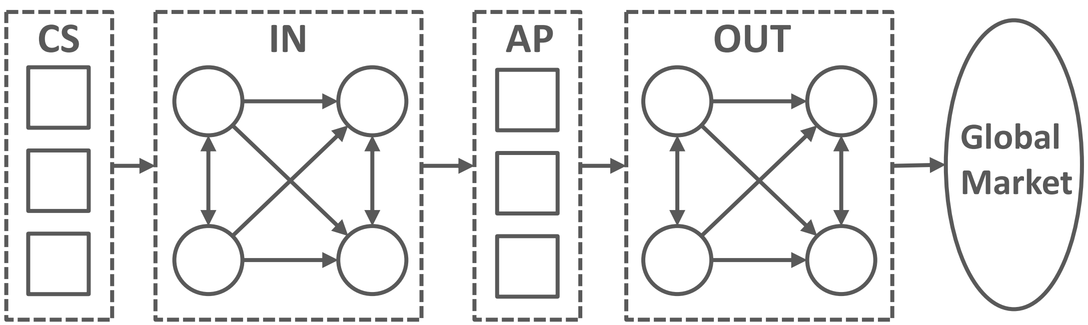
Introduction

Quantitative Models for SCM – Taxonomy:

Physical environment

- Stages
  - One-stage, two-stage, multi-stage
- Stages considered in the Supply Chain structure:
  - Component Supplier (CS)
  - Inbound Logistics (IN)
  - Assembly Plants (AP)
  - Outbound Logistics (OUT)

Global Supply Chain Model



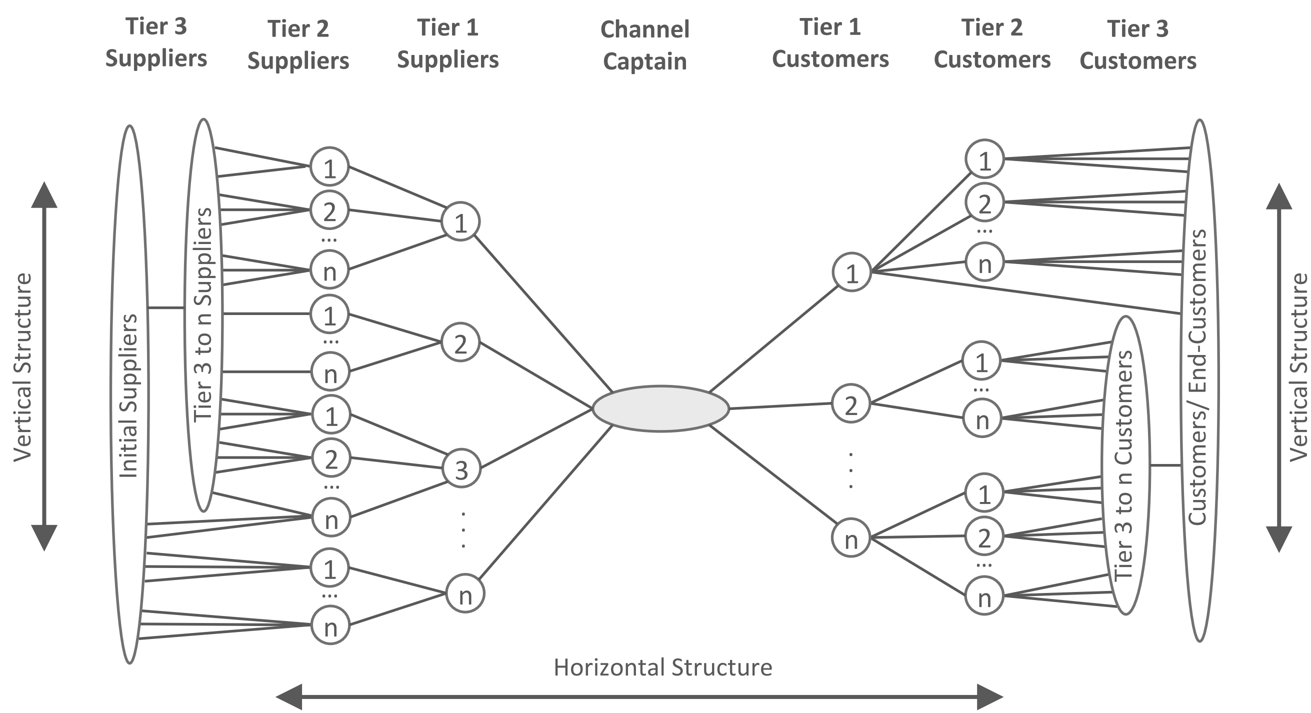
Pontrandolfo, P. & Okogbaa, O. (1999). Global manufacturing: a review and a framework for planning in a global corporation. *International Journal of Production Research*, 37(1), 1-19.

Introduction

Quantitative Models for SCM – Taxonomy:

Physical environment

- Layers



Lambert, D. M., Cooper, M. C., & Pagh, J. D. (1998). Supply chain management: implementation issues and research opportunities. *The international journal of logistics management*, 9(2), 1-20.

Introduction

- Quantitative Models for SCM – Taxonomy:
  - **Modelling approach**
    - Deterministic models
      - →All model parameters are fixed and known with certainty
    - Stochastic models
      - →Some model parameters are uncertain or random
    - Hybrid models
      - →Mixed models (deterministic and stochastic)
    - IT-driven models
      - →Try to integrate and coordinate various stages of Supply Chain planning on a real time basis and to improve visibility through the Supply Chain using software applications

Min, H., & Zhou, G. (2002). Supply chain modeling: past, present and future. *Computers & industrial engineering*, 43(1), 231-249.

Introduction

- Quantitative Models for SCM – Taxonomy:
  - **Modelling approach**
    - Deterministic models
      - Single objective
      - Multiple objectives
    - Stochastic models
      - Optimal Control Theory
      - Dynamic Programming
    - Hybrid models
      - Inventory Theoretic
      - Simulation
    - IT-driven models
      - CPFR (Collaborative Planning and Forecasting Replenishment)
      - MRP (Materials Requirement Planning)
      - DRP (Distribution Resource Planning)
      - ERP (Enterprise Resource Planning)
      - GIS (Geographic Information System)

Introduction

- Quantitative Models for SCM: Centralized vs Distributed
  - **Centralized decision making**
    - A single decision-maker that has authority to manage the operations of all entities of the Supply Chain
      - Centralizes all the needed information necessary for decision making
      - Takes decisions on the proper operation of the SC as a whole
      - Based on some agreed objectives of the SC partners
      - A single quantitative model
  - **Distributed decision making**
    - Several decision-makers
      - Each decider makes its own plans based on:
        - Own objectives and constrains
        - Some private information
      - As much quantitative models as as decision-makers
      - Need for coordination of all quantitative models

Which will achieve the optimum for all the Supply Chain?

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Mathematical Programming Models

Mathematical Programming

- In Operations Research, Mathematical Programming is the selection of a best element (with regard to some criteria) from some set of available alternatives.
- Main types:
  - Linear Programming (LP)
    - Continuous Linear Programming (CLP)
    - Integer Linear Programming (ILP)
    - Mixed Integer Linear Programming (MILP)
  - Non-Linear Programming (NLP)
    - Continuous Non-Linear Programming (CNLP)
    - Integer Non-Linear Programming (INLP)
    - Mixed Integer Non-Linear Programming (MINLP)
  - Quadratic Programming (QP)

Mathematical Programming Models

- Structure
  - Indexes
    - Model elements (factory, machine, part, etc.)
  - Sets
    - Groups of instances of indexes (set of machines, etc.)
  - Parameters, data
    - Known attributes which can not be changed (cost, demand, etc.)
  - Decision variables
    - Unknown attributes which can be changed (amount to produce, etc.)
  - Objective(s)
    - Value(s) which the decision-maker wants to optimize (maximize or minimize)
  - Constraints
    - Limits to satisfy (resources capacity, available money, etc.)

Mathematical Programming Models

A simple example

- Nitron Corporation manufactures 2 products (A and B) using 2 machines (P and Q). Product A provides a benefit of 60€ per unit, product B provides a benefit of 50€ per unit. Each unit of product A requires 10 min of machine P and 8 min of machine Q. Each unit of product B requires 20 min of machine P and 5 min of machine Q. Machine P capacity is 200 min per day. Machine Q capacity is 80 min per day. The minimum production should be 2 units of A and 5 units of B per day. ¿How many units A and B should Nitron produce per day?

Mathematical Programming Models

- Indexes:

  i : products

  j : machines

- Sets:

  i : {A, B}

  j : {P, Q}

- Data:

  Be_i : [60, 50] → Be_A = 60 ; Be_B = 50

  Mp_i : [2, 5] → Mp_A = 2 ; Mp_B = 5

  Ca_j : [200, 80] → Ca_P = 200 ; Ca_Q = 80

  Req_{ij} : [[10, 8] [20, 5]] → Req_{AP} = 10 ; Req_{AQ} = 8 ; Req_{BP} = 20 ; Req_{BQ} = 5

- Decision variables:

  X_i → X_A ; X_B

Mathematical Programming Models

- Objective:

  Maximize benefit $$MaxZ = \sum_{i} Be_i \cdot X_i$$

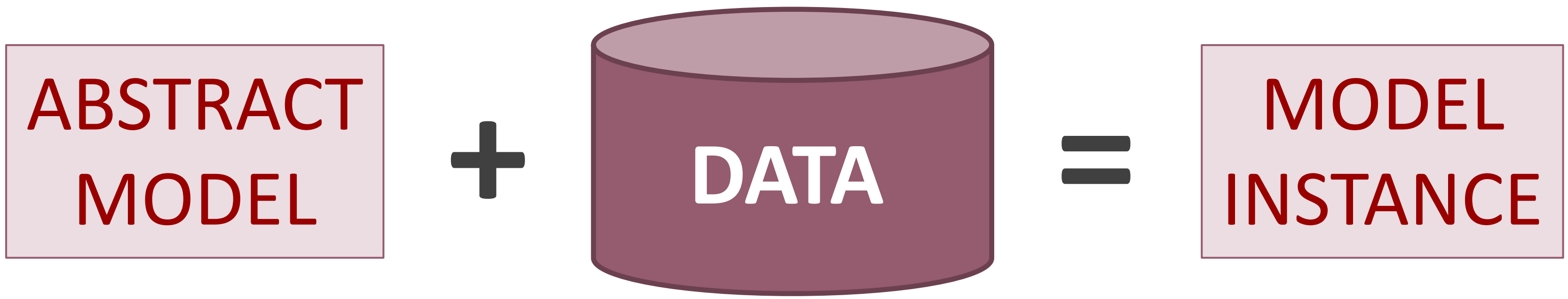
- Constraints:

  Machines capacity $$\sum_{i} Req_{ij} \cdot X_i \leq Ca_j \qquad j = \{P, Q\}$$

  Minimum production $$X_i \geq Mp_i \qquad i = \{A, B\}$$

ABSTRACT MODEL

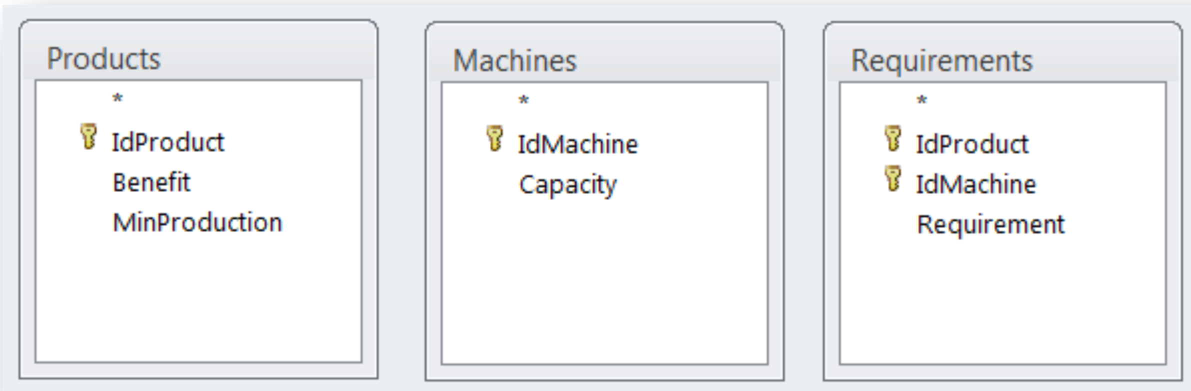
Mathematical Programming Models



- Independency of data
- Solve different problems when data changes
- Scalability
  - 2 products → 100 products
  - 2 machines → 50 machines

Mathematical Programming Models

- Data stored in a DATABASE



Products

IdProduct	Benefit	MinProduction	Production
A	50	2	
B	60	5	

Requirements

IdProduct	IdMachine	Requirement
A	P	10
A	Q	8
B	P	20
B	Q	5

Machines

IdMachine	Capacity
P	200
Q	80

Mathematical Programming Models

- Model written with an Algebraic Modelling Language (AML)
  - AMPL
    - http://ampl.com/
  - GAMS (General Algebraic Modelling System)
    - http://gams.com/
  - LINDO/LINGO
    - http://www.lindo.com/
  - MPL (Mathematical Programming Language)
    - http://maximalsoftware.com/
  - Pyomo
    - http://www.pyomo.org/
  - JuMP (Julia for Mathematical Programming)
    - http://www.juliaopt.org/ https://github.com/JuliaOpt/JuMP.jl

Mathematical Programming Models

Classwork → model in MPL

- Create the tables in the **Nitron.mdb** database and fill the data
- Create the **Nitron.mpl** model
- Obtain the solution

Products
- *
- IdProduct
- Benefit
- MinProduction

Machines
- *
- IdMachine
- Capacity

Requirements
- *
- IdProduct
- IdMachine
- Requirement

Products

IdProduct	Benefit	MinProduction	Production
A	50	2	
B	60	5	

Requirements

IdProduct	IdMachine	Requirement
A	P	10
A	Q	8
B	P	20
B	Q	5

Machines

IdMachine	Capacity
P	200
Q	80

Mathematical Programming Models

- Model sections in MPL

```
TITLE

OPTIONS

INDEX

DATA

VARIABLES

MACROS

MODEL

SUBJECT TO

BOUNDS

INTEGER

BINARY

FREE

END
```

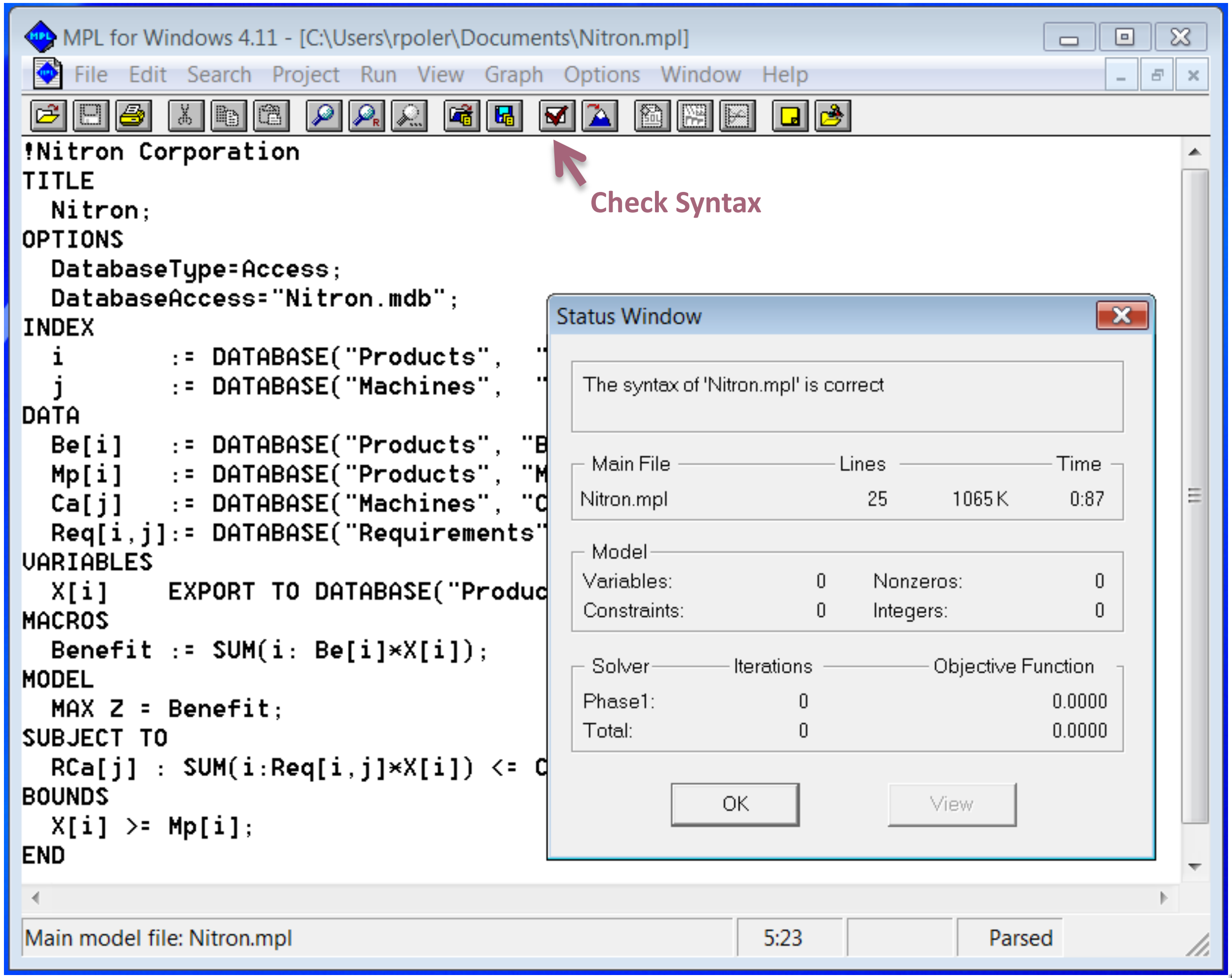
Mathematical Programming Models

- Model in MPL

```
!Nitron Corporation
TITLE
  Nitron;
OPTIONS
  DatabaseType=Access;
  DatabaseAccess="Nitron.mdb";
INDEX
  i         := DATABASE("Products",   "IdProduct");
  j         := DATABASE("Machines",   "IdMachine");
DATA
  Be[i]     := DATABASE("Products", "Benefit");
  Mp[i]     := DATABASE("Products", "MinProduction");
  Ca[j]     := DATABASE("Machines", "Capacity");
  Req[i,j]:= DATABASE("Requirements", "Requirement");
VARIABLES
  X[i]      EXPORT TO DATABASE("Products" , "Production");
MACROS
  Benefit := SUM(i: Be[i]*X[i]);
MODEL
  MAX Z = Benefit;
SUBJECT TO
  RCa[j] : SUM(i:Req[i,j]*X[i]) <= Ca[j];
BOUNDS
  X[i] >= Mp[i];
END
```

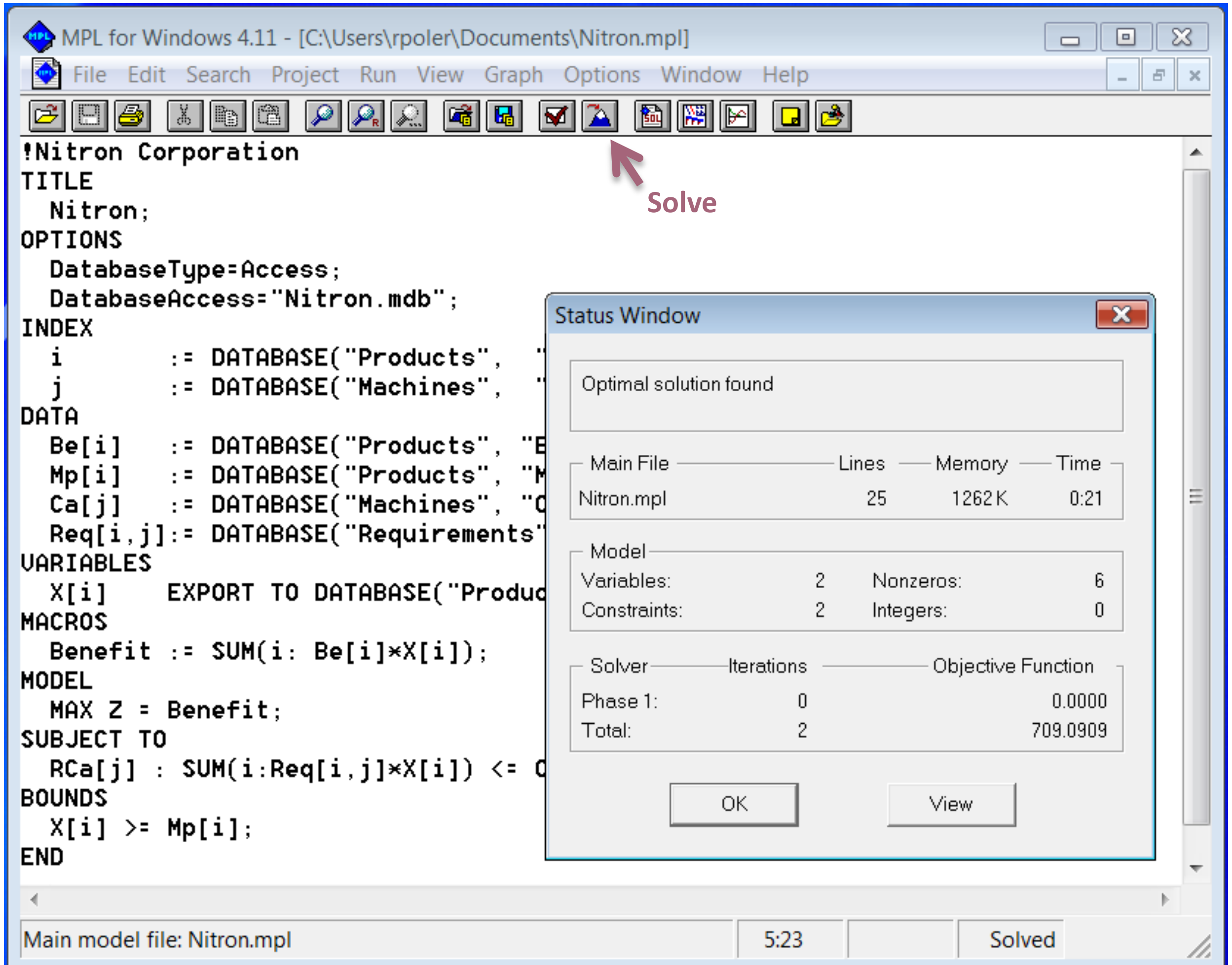
Mathematical Programming Models

- Model check in MPL



Mathematical Programming Models

- Model solve in MPL



Mathematical Programming Models

- MPL solution

```
SOLUTION RESULT
  Optimal solution found
    MAX Z        =        709.0909
MACROS
    Macro Name                          Values
  -----------------------------------------------
    Benefit                           709.0909
  -----------------------------------------------
DECISION VARIABLES
VARIABLE X[i] :
   i           Activity     Reduced Cost
  ---------------------------------------
   A             5.4545           0.0000
   B             7.2727           0.0000
  ---------------------------------------
CONSTRAINTS
CONSTRAINT RCa[j] :
   j              Slack     Shadow Price
  ---------------------------------------
   P             0.0000           2.0909
   Q             0.0000           3.6364
  ---------------------------------------
END
```

Mathematical Programming Models

- Solution in database

Products

IdProduct	Benefit	MinProduction	Production
A	50	2	5,4545454545
B	60	5	7,2727272727

Mathematical Programming Models

- Model in MPL (integer values)

```
TITLE
  Nitron;
OPTIONS
    DatabaseType=Access;
    DatabaseAccess="Nitron.mdb";
INDEX
  i        := DATABASE("Products",  "IdProduct");
  j        := DATABASE("Machines",  "IdMachine");
DATA
  Be[i]    := DATABASE("Products", "Benefit");
  Mp[i]    := DATABASE("Products", "MinProduction");
  Ca[j]    := DATABASE("Machines", "Capacity");
  Req[i,j]:= DATABASE("Requirements", "Requirement");
VARIABLES
  X[i]      EXPORT TO DATABASE("Products" , "Production");
MACROS
  Benefit := SUM(i: Be[i]*X[i]);
MODEL
  MAX Z = Benefit;
SUBJECT TO
  RCa[j] : SUM(i:Req[i,j]*X[i]) <= Ca[j];
BOUNDS
  X[i] >= Mp[i];
INTEGER
  X[i];
END
```

Mathematical Programming Models

- MPL solution (integer values)

```
SOLUTION RESULT
  Optimal integer solution found
    MAX Z        =        680.0000
MACROS
    Macro Name                            Values
  -----------------------------------------------
    Benefit                             680.0000
  -----------------------------------------------
DECISION VARIABLES
VARIABLE X[i] :
   i            Activity     Reduced Cost
  ---------------------------------------
   A              4.0000           0.0000
   B              8.0000         -40.0000
  ---------------------------------------
CONSTRAINTS
CONSTRAINT RCa[j] :
   j               Slack     Shadow Price
  ---------------------------------------
   P              0.0000           5.0000
   Q              8.0000           0.0000
  ---------------------------------------
END
```

Mathematical Programming Models

- Solution in database (integer values)

Products

IdProduct	Benefit	MinProduction	Production
A	50	2	4
B	60	5	8

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Strategic Models for Supply Chain Design

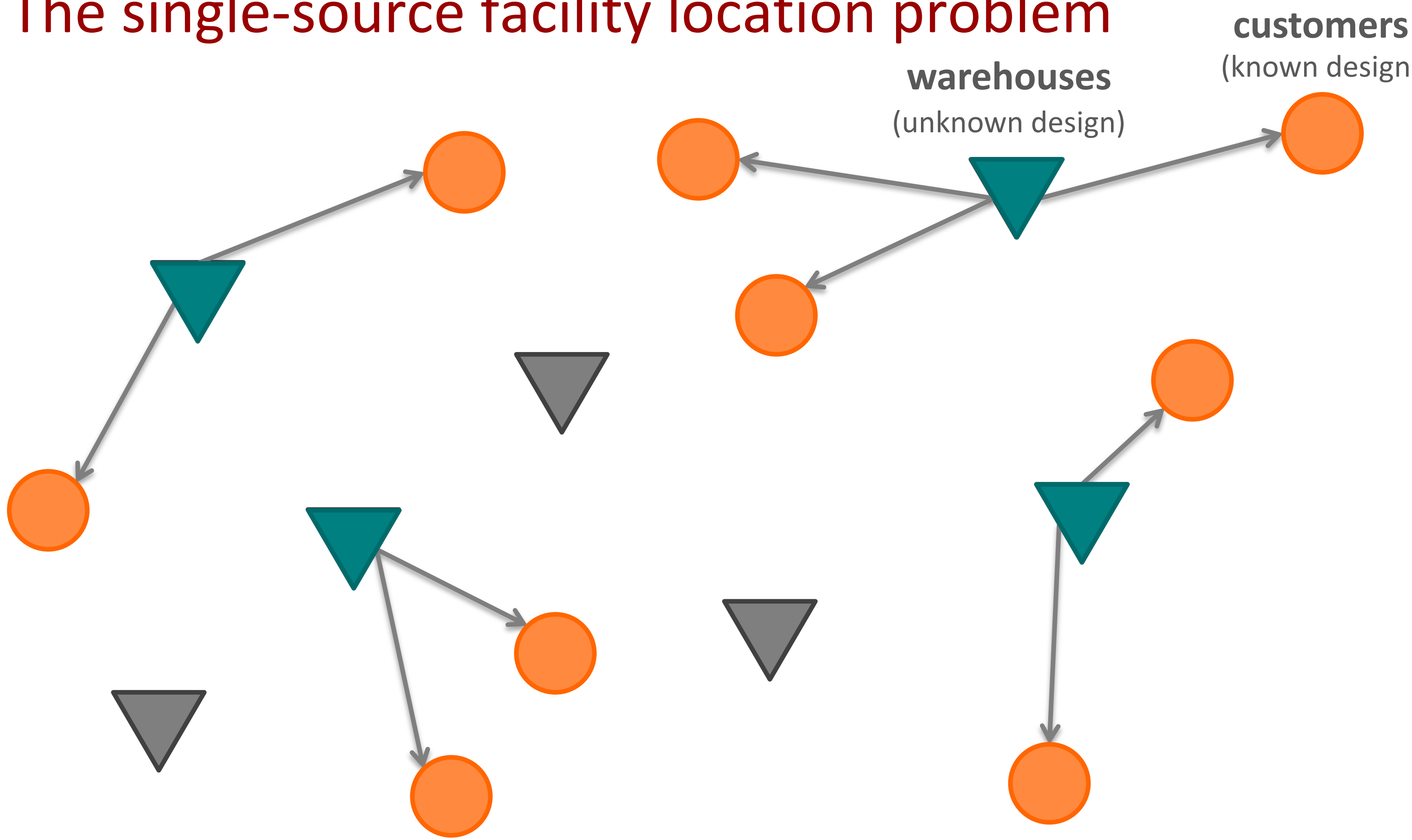
The single-source facility location problem

- Strategic / One-stage / Deterministic
  - Goal: locating a set of warehouses in a distribution network
    - Retailers geographically dispersed in a region
    - There are m preselected locations as possible store locations
    - Retailers want to receive products from a single warehouse
  - The cost of placing a warehouse at a particular location includes
    - Fixed Cost: construction costs, maintenance, etc.
    - Variable costs: transport costs
  - Decision variable: locations where to locate the warehouses
  - Objective: minimize the total cost

Muriel, A., & Simchi-Levi, D. (2003). Supply chain design and planning–applications of optimization techniques for strategic and tactical models. *Handbooks in operations research and management science*, 11, 15-93.

Strategic Models for Supply Chain Design

- The single-source facility location problem



Strategic Models for Supply Chain Design

The single-source facility location problem

- Indexes:
  - i : retailers
  - j : locations (in which place the warehouses)
- Data:
  - d_i : yearly demand from retailer i
  - b_{ij} : cost of transporting d_i units from warehouse j to retailer i
  - F_j : yearly operation cost of warehouse j
  - q_j : capacity of warehouse j in units
- Decision variables:
  - Y_j : binary { 1 if a warehouse is placed in location j } { 0 otherwise }
  - X_{ij} : binary { 1 if the warehouse j supplies the retailer i } { 0 otherwise }

Strategic Models for Supply Chain Design

The single-source facility location problem

- Objective:

  Minimize transport cost and operation cost

$$MinZ = \sum_i \sum_j b_{ij} \cdot X_{ij} + \sum_j F_j \cdot Y_j$$

- Constraints:

  Units transported from a warehouse ***j*** to all the retailers ***i*** (to which it supplies) should be less than its capacity

$$\sum_i d_i \cdot X_{ij} \leq q_j \cdot Y_j \qquad \forall j$$

  Retailers should receive products from a single warehouse

$$\sum_j X_{ij} \leq 1 \qquad \forall i$$

Strategic Models for Supply Chain Design

The single-source facility location problem

- Constraints: (cont)

  Demand from all the retailers ***i*** should be fulfilled

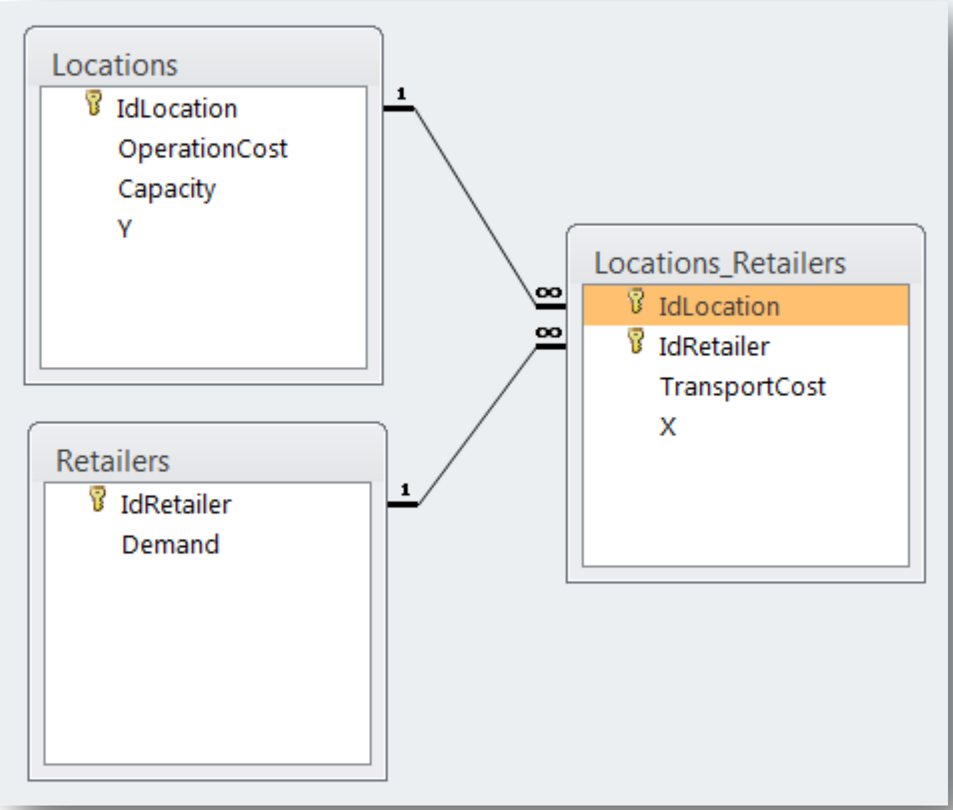
$$\sum_j d_i \cdot X_{ij} \geq d_i \qquad \forall i$$

Strategic Models for Supply Chain Design

The single-source facility location problem

Classwork → model in MPL

- Use the **SSFLP.mdb** database
- Create the **SSFLP.mpl** model
- Obtain the solution



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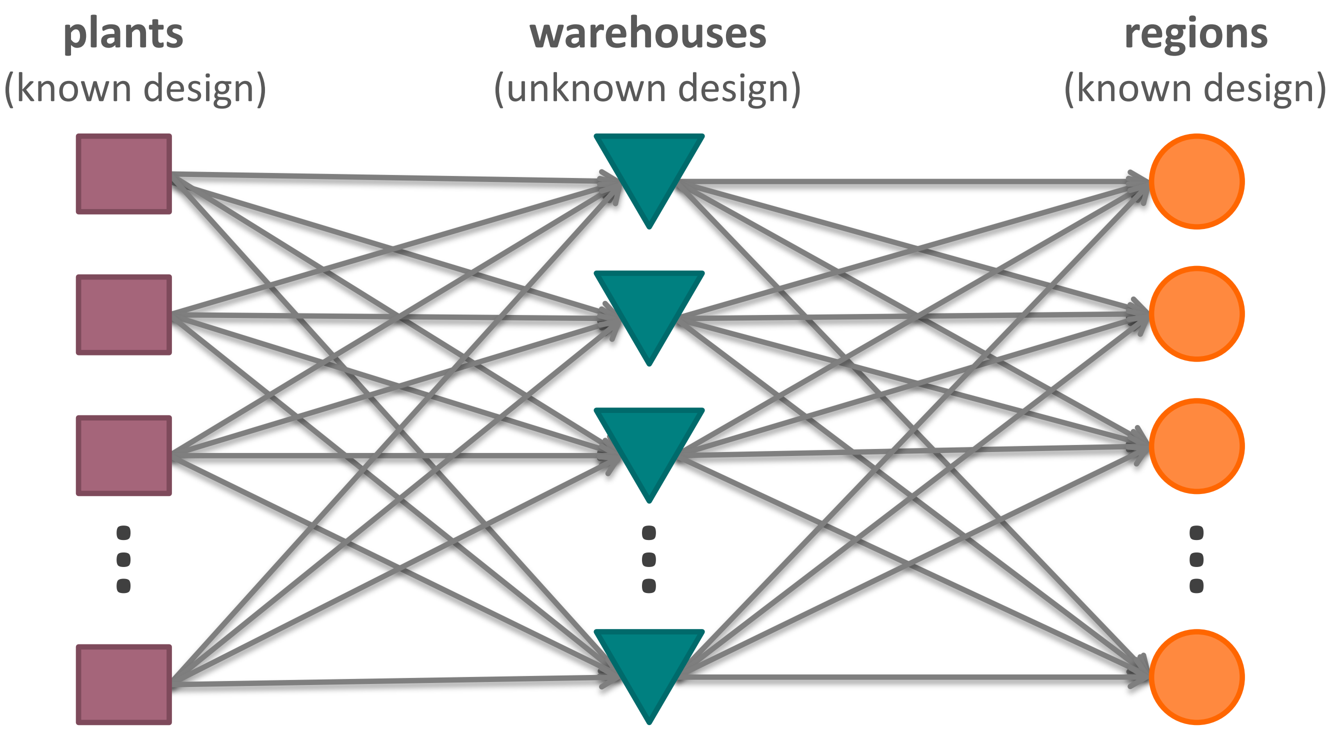
Strategic Models for Supply Chain Design

The distribution system problem

- Strategic / One-stage / Deterministic
  - Goal: define the optimum network of distribution warehouses for distributing products to retailers (regions) from production plants
    - Production plants are known (amount and location)
    - Retailers are known and grouped in regions
    - Warehouses should be built in pertinent locations
  - Costs:
    - Fixed Cost: warehouses construction; warehouses operation
    - Variable costs: transport costs between production plants and warehouses and between warehouses and regions; warehouses maintenance
  - Decision variables:
    - locations where to locate the warehouses; warehouses assignment to regions; amount of products transported from plants to warehouses and from warehouses to regions
  - Objective: minimize the total cost

Strategic Models for Supply Chain Design

The distribution system problem



Miller, T. (2002). Distribution And Transportation Planning And Scheduling. In *Hierarchical Operations and Supply Chain Planning* (pp. 95-158). Springer London.

Strategic Models for Supply Chain Design

The distribution system problem

- Indexes:
  - i : production plants
  - j : warehouses
  - k : regions (of retailers)
  - l : products
- Data:
  - d_{kl} : demand from region k of product l
  - a_{ijl} : cost of transporting 1 unit of product l from plant i to warehouse j
  - b_{jkl} : cost of transporting 1 unit of product l from warehouse j to region k
  - I_j : cost of building warehouse j
  - F_j : yearly operation cost of warehouse j
  - v_{jl} : handling cost of 1 unit of product l in warehouse j
  - c_{il} : yearly production capacity of product l in plant i
  - C : maximum amount of warehouses to build
  - B : maximum investment for warehouses building

Strategic Models for Supply Chain Design

The distribution system problem

- Decision variables:

  Y_j : binary { 1 if a warehouse j is built} { 0 otherwise }

  W_{jk} : binary { 1 if the warehouse j supplies the region k } { 0 otherwise }

  S_{ijl} : units of product l transported from plant i to warehouse j

  T_{jkl} : units of product l transported from warehouse j to region k

- Objective:

  Minimize transport cost, handling cost and building and operation cost

$$MinZ = \sum_i \sum_j \sum_l a_{ijl} \cdot S_{ijl} + \sum_j \sum_k \sum_l b_{jkl} \cdot T_{jkl} +$$

$$\sum_j \sum_k \sum_l d_{kl} \cdot v_{jl} \cdot W_{jk} + \sum_j \left(F_j \cdot Y_j + I_j \cdot Y_j \right)$$

Strategic Models for Supply Chain Design

- Constraints:

Material flows

The demand of all products from all regions should be satisfied

$$\sum_{j} T_{jkl} = d_{kl} \qquad \forall k, \forall l$$

The amount of each product which arrives to a warehouse should be equal to the amount which exit from that warehouse

$$\sum_{i} S_{ijl} = \sum_{k} T_{jkl} \qquad \forall j, \forall l$$

Physical resources limitations

The amount of each product produced by a plant should not exceed the production capacity

$$\sum_{j} S_{ijl} \leq c_{il} \qquad \forall i, \forall l$$

Strategic Models for Supply Chain Design

The distribution system problem

- Constraints: (cont)

Financial resources limitations

The amount of money invested in building warehouses should not exceed the investment budget

$$\sum_j I_j \cdot Y_j \leq B$$

Company policies

Each region receives all its products from only one warehouse

$$\sum_j W_{jk} = 1 \qquad \forall k$$

The number of warehouses built should not exceed the limit

$$\sum_j Y_j \leq C$$

Strategic Models for Supply Chain Design

- Constraints: (cont)

Logical constraints

A warehouse will supply a region only when the amount transported of all products from such warehouse to such region is nonzero

$$\sum_{l} T_{jkl} \leq M \cdot W_{jk} \qquad \forall j, \forall k$$

this is a large number

A warehouse should be built if it supplies to any region

$$W_{jk} \leq Y_j \qquad \forall j, \forall k$$

Strategic Models for Supply Chain Design

The distribution system problem

Classwork → model in MPL

- Use the **DSP.mdb** database
- Create the **DSP.mpl** model
- Obtain the solution