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Production Scheduling in a vegetable packing machine with uncertainty in the quality of the raw material.

WP8 Agri-food supply chain decision-making under uncertainty

Teaching Sesion

Pedro Gómez Gasquet

Secondment in Riviera, July 2019

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Index

1. Problem Description
2. MILP Model A: Basic
3. MILP Model B: Raw Material Yield Considered
4. Solve the problem.
5. Next Steps.

Problem Description

- **Focus on Production Scheduling Problem**

- **Demand Planning, Raw Material Planning or Distribution Planning are out of our scope. Demand, Available Material (vegetables), etc. are not decision variables.**

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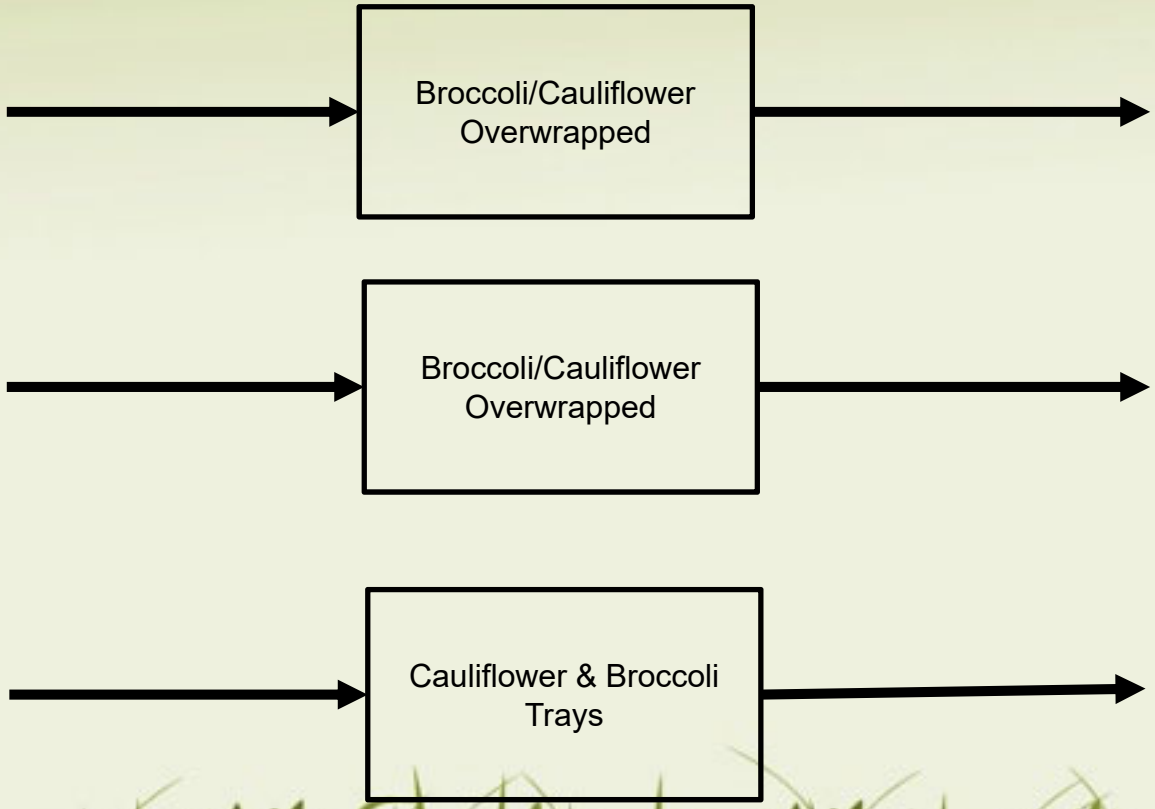
Problem Description

- **Lets consider the existence of a productive system with 3 work centers.**



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Problem Description

- **Broccoli/Cauliflower Work Centre**
  - Family of products associated with the broccoli/Cauliflower overwrapped is elaborated.
  - Products in a family are differentiated by its weight, e.g.400 gr.), B (360 gr.) Or C (350 gr.).
  - Formed by two conveyor belts in which the broccoli(Cauliflower is placed individually and on each one they carry out different operations (cleaning, bagging, weighing, and boxing).
  - The system works as if they were two unrelated resources in parallel from the production scheduling point of view.

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Problem Description

- **Broccoli/Cauliflower Work Centre**
  - Raw material presents uncertainty in terms of its weight, so it is not possible to know precisely how many broccoli/cauliflower must be used to obtain a final product of a certain quality.
  - The product that does not adjust to the desired weight will remove the circuit and will be treated as non-compliant.
  - The non-conforming product can be reused for another type of product in this same work centre (same family) or to make another product such as the mix of cauliflower and broccoli trays.

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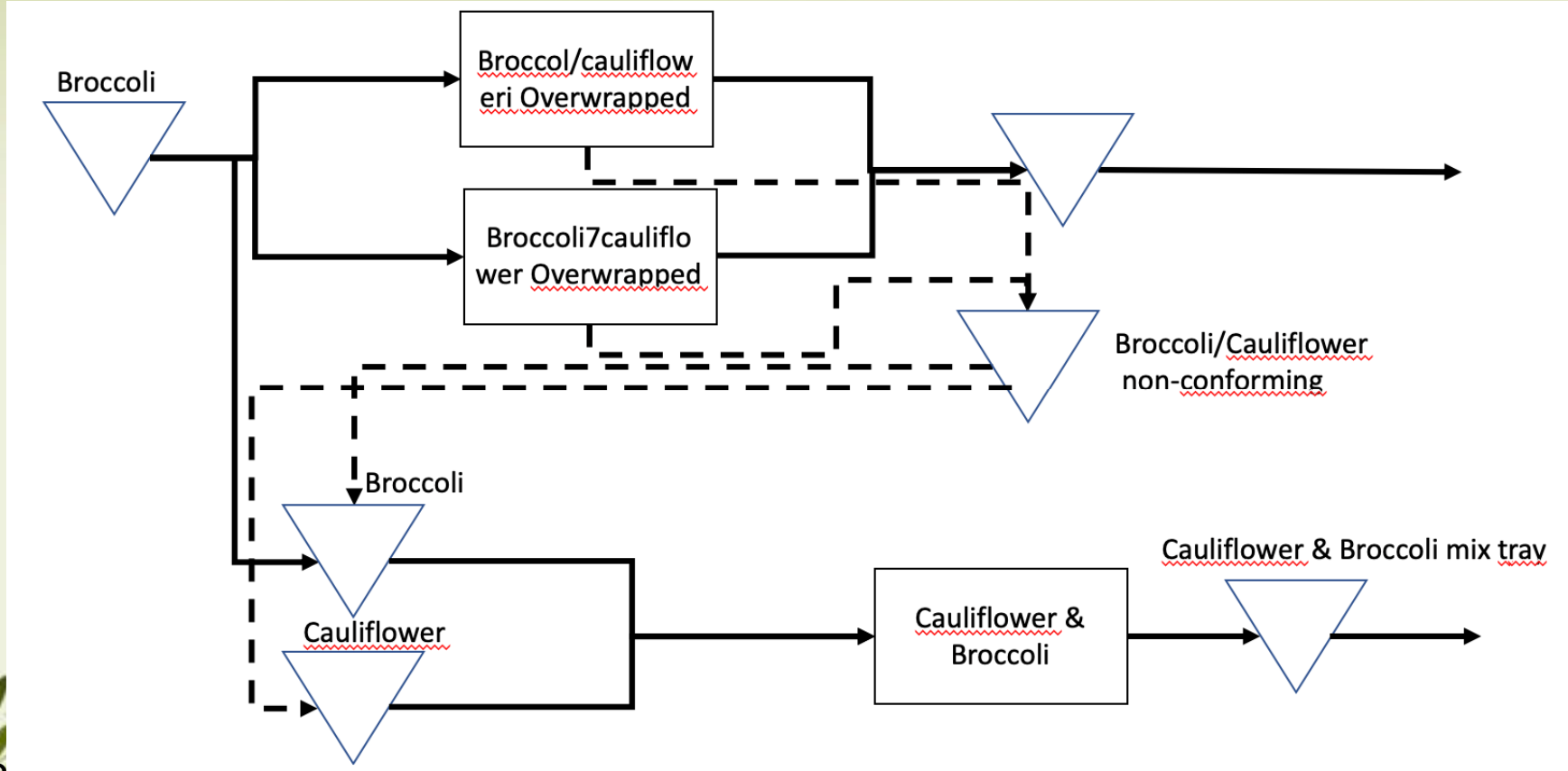
Problem Description

- **Broccoli&Cauliflower Mix Trays Work Centre**
  - Product is elaborated whose raw material is the broccoli and the natural cauliflower in florets.
  - Both materials are packaged separately in the same plastic tray.
  - Formed by a conveyor belt in which the broccoli and the cauliflower are placed individually in florets.
  - Several operations are carried out (cleaning, packaging, etc.).
  - The system works as if it were a single resource from the point of view of Production Scheduling.

Problem Description

- **Lets consider the existence of a productive system with 3 work centres.**



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Problem Description

- **Hypothesis:**
  - It be assumed that there is an adjusted planning of the harvests and that both the cauliflower and the broccoli are available in a certain amount at the beginning of each day.
  - A single period will be considered, e.g an 8-hour work day every day.
  - Products could have different shelf-life determined by the farmer's delivery date and the time waiting in the warehouse.
  - Each day that the product is company loses value which is reflected in an economic penalty.
  - A milk-run distribution is considered. Due dates are not considered.

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Model A: Basic

The problem is to decide the sequencing of jobs and the amount of raw material used in each product demanded to maximize the benefits.

As a result of the resolution of the model, the dates of completion of each job are also determined and, consequently, the start dates could be calculated in a simple way.

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Model A: Basic

The information will be presented using the following index:

- i, l Index of orders set N {1..n}
- j Index of resources set M {1..m}
- t Index of Broccoli quality set Q {1..q}
- tt Index of Cauliflower quality set QQ {1..qq}

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Model A: Basic

The parameters of the model are:

- Qb_t (integer) kg. of broccoli available at the beginning of the day for quality t.
- Qc_{tt} (integer) kg. of cauliflower available at the beginning of the day for quality tt.
- D_i (integer) kg. required for the product i.
- I_i (integer) Expected income for the product i.
- L_i (integer) Shelf-life available for the product i (hours).
- W_i (integer) Waiting hours used for the product i.
- PY_i (integer) Lost of shelf-life economic penalty for the product i.
- PTY_i (integer) Unattended demand economic penalty for the product i.

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Model A: Basic

The parameters of the model are:

- BC_t (real) Raw broccoli cost of quality t.
- CC_{tt} (real) Raw cauliflower cost of quality tt.
- $BR_{t,i}$ (boolean) 1 if quality of broccoli t is conform for the product i.
- $CR_{tt,i}$ (boolean) 1 if quality t of cauliflower is conform for the product i.
- $WC_{j,i}$ (boolean) 1 if work centre j can process product i.
- $P_{j,i}$ (integer) Processing time of the product i in the work centre j.
- M (integer) Large number.

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Model A: Basic

The MILP model determines the value of the following variables:

- $xb_{t,i}$ (integer) broccoli (kg) of quality t assigned to the product i.
- $xc_{t,i}$ (integer) cauliflower (kg) of quality t assigned to the product i.
- $wc_{j,i}$ (boolean) 1 if product i is processed in the work centre j, otherwise 0.
- k_i kg. of the product i in the lot.
- $y_{i,l}$ (boolean) 1 if product I is processed before product l.
- c_i (integer) completion time for the product i.
- u_i (integer) Shelf-life available at completion time for the product i.

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Model A: Basic

The objective is to maximize the benefit :

$$F.O.\ max\ z = \{\sum_{i=1}^{n}\Big(q_i * I_i - (D_i - k_i) * PTY_i - u_i * PY_i - \Big(\sum_{t=1}^{q}(xb_{t,i} * BC_t) + \sum_{tt=1}^{qq}(xc_{tt,i} * CC_{tt})\Big)\Big)\}$$

Expected revenues

Unattended demand penalty

Shelf-life reduction penalty

Cost of product (broccoli & Cauliflower)

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Model A: Basic

The constraints of the model are presented below in two sets (2-5 and 6-13), each representing a type of system restriction.

The model is subject to:

$$Qb_t \geq \sum_{i=1}^{n} xb_{t,i} \quad \forall t \qquad (2)$$

$$Qc_{tt} \geq \sum_{i=1}^{n} xb_{tt,i} \quad \forall tt \qquad (3)$$

Constraints 2 and 3 prevent more raw material from being consumed than is available.

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Model A: Basic

The constraints 2-5 are related to product quantity and quality. The model is subject to:

$$D_i \geq k_i \quad \forall i \qquad (4)$$

Constrain 4 does not allow more final product to be prepared than demanded. Although it allows not to meet a demand.

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Model A: Basic

The constraints of the model are presented below in two sets (2-5 and 6-13), each representing a type of system restriction. The model is subject to:

$$k_i = \sum_{t=1}^{q} xb_{t,i} \quad * BR_{t,i} + \sum_{tt=1}^{qq} xc_{tt,i} \quad * CR_{tt,i} \quad \forall i (5)$$

Constrain 5 relates the quantity of final product with the raw material used. This relationship allows using any quality that is allowed to process a determined final product.

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Model A: Basic

The constraints 6-13 are related to completion times. The model is subject to:

$$\sum_{j=1}^{m} wc_{j,i} = 1 \quad \forall i \qquad (6)$$

$$\sum_{j=1}^{m} wc_{j,i} * WC_{j,i} = 1 \quad \forall i \qquad (7)$$

Constraint 6 requires that all orders be assigned to a work centre. And 7 restricts to 6, forcing it to be one of the work centres where the product can be processed.

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Model A: Basic

The constraints 6-13 are related to completion times. The model is subject to:

$$w_{j,i} + \sum_{r=1/r\neq j}^{m} w_{r,l} + y_{i,l} \leq 2 \quad \forall i \; \forall l \; \forall j (8)$$

Restriction 8 forces the variable $y_{i,l}$ can only be 1 when jobs i and l are assigned to the same work centre.

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Model A: Basic

The constraints 6-13 are related to completion times. The model is subject to:

$$c_l - c_i + \mathrm{M} * (3 - \mathrm{y}_{i,l} - w_{j,i} - w_{j,l}) \geq P_{j,l} * \mathrm{k}_l \quad \forall i, \forall l, \forall j \tag{9}$$

$$c_i - c_l + \mathrm{M} * (2 + \mathrm{y}_{i,l} - w_{j,i} - w_{j,l}) \geq P_{j,i} * \mathrm{k}_i \quad \forall i, \forall l, \forall j \tag{10}$$

Constraint 9 and 10 force the completion times of the jobs to correspond to the order of the sequence.

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Model A: Basic

The constraints 6-13 are related to completion times. The model is subject to:

$$c_l - c_i + \mathrm{M} * (3 - \mathrm{y}_{i,l} - w_{j,i} - w_{j,l}) \geq P_{j,l} * \mathrm{k}_l \quad \forall i, \forall l, \forall j \qquad (9)$$

$$c_i - c_l + \mathrm{M} * (2 + \mathrm{y}_{i,l} - w_{j,i} - w_{j,l}) \geq P_{j,i} * \mathrm{k}_i \quad \forall i, \forall l, \forall j \qquad (10)$$

Constraint 9 is aimed at forcing the completion times when two jobs i and l are assigned to the same center and job i is sequenced before l.

With constraint 9, the solver could assign value 0 to any element in the set, since it would satisfy the equation. To avoid this, the restriction 10 forces that when a zero is assigned to the variable yi, l it is satisfied that the completion time of l is at least that of i plus the processing time of l.

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Model A: Basic

The constraints 6-13 are related to completion times. The model is subject to:

$$c_i + \mathrm{M} * \left(1 - \mathrm{wc}_{j,i}\right) \geq \quad P_{j,i} * \mathrm{k}_i \quad \forall i \; \forall j \qquad (11)$$

$$\left(L_i - W_i - c_i\right) \leq \; u_i \qquad (12)$$

$$u_i \geq 0 \qquad (13)$$

Constraint 11, the completion times of the first jobs of each work centre are at least equal to the time of their processing time. Constraints 12 and 13 allow associating the variable with the shelf-life hours of the product once the waiting and process times have been subtracted. This value will never be negative.

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Model B:

Raw Material Yield Considered

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Model B: Raw Material Yield Considered

The lack of conformity in the quality of the raw material is introduced. This is understood as the fact that the number of kg of broccoli or cauliflower that are in accordance with the type of product requested from the farmer is not known precisely.

This second model, uncertainty will be introduced in a very simple way, as a first step.

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Model B: Raw Material Yield Considered

- With sets Q and QQ the types of possible qualities of broccoli and cauliflower are modelled.
- The quality of the raw material delivered from in a crop harvested as quality t can be composed by several percentages of different qualities.
- Triangular Matrix QBM include the percentage of expected raw material of t quality and all the lower ones for broccoli. (QCM for cauliflower). The yield of raw material for a final product.
- A product conforms to its quality if it uses raw material of the same or higher quality, which was already reflected in the BR and CR matrices.

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Model B: Raw Material Yield Considered

An example of Q set:

Q	1(A)	2 (B)	3 (C)

An example of Triangular matrix QBM:

Real quality expected

Quality of raw material delivered

QBM	1(A)	2 (B)	3 (C)
1 (A)	95%	4%	1%
2 (B)	0%	97%	3%
3 (C)	0%	0%	100%

An example of BR matrix:

Order (final product)

Quality of raw material needed

BR	Order 1 (QA)	Order 2 (QB)	Order 3 (QC)	Order 4 (TX)
1 (A)	1	1	1	1
2 (B)	0	1	1	1
3 (C)	0	0	1	1

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Model B: Raw Material Yield Considered

An example of the relationship between final product (broccoli overwrapped) and raw broccoli.

Q	1(A)	2 (B)	3 (C)

QBM	1(A)	2 (B)	3 (C)
1 (A)	95%	4%	1%
2 (B)	0%	97%	3%
3 (C)	0%	0%	100%

BR	Order 1 (QA)	Order 2 (QB)	Order 3 (QC)	Order 4 (TX)
1 (A)	1	1	1	1
2 (B)	0	1	1	1
3 (C)	0	0	1	1

If model use $xb_{1,2}$=100kg of QA broccoli for order 2 (QB Product) then 99 Kg are conform.

100*0,95*1+100*0,04*1+100*0,01*0 = 99 Kg

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Model B: Raw Material Yield Considered

Constraint 5 should be modified so that the calculation of Ki reflects the performance according to the quality of the raw material used and the desired final product:

$$k_i = \sum_{t=1}^{q} xb_{t,i} * BR_{t,i} + \sum_{tt=1}^{qq} xc_{tt,i} * CR_{tt,i} \quad \forall i \qquad (5)$$

$$k_i = \sum_{t=1}^{q} \sum_{t'}^{q} \left(xb_{t,i} * QBM_{t,t'} * BR_{t',i} \right) + \sum_{tt=1}^{qq} \sum_{tt'}^{q} \left(xc_{tt,i} * QCM_{tt,tt'} * CR_{tt',i} \right) \quad \forall i \qquad (5')$$

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Model B: Raw Material Yield Considered

The objective function should include the cost of storage of the raw material that is not used during the current period but could be used in the next. CT_i is the storage cost of the raw material needed for the product i.

$$F.O.\ max\ z = \{\sum_{i=1}^{n}\left(\mathrm{q}_i * I_i - (D_i - k_i) * PTY_i - u_i * PY_i - \left(\sum_{t=1}^{q}(xb_{t,i} * BC_t) + \sum_{tt=1}^{qq}(xc_{tt,i} * CC_{tt})\right) - \left(\sum_{t=1}^{q} xb_{t,i} + \sum_{tt=1}^{qq} xc_{tt,i} - k_i\right) * CT_i)\right\} \quad (1')$$

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Solve the Problem

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Solve the problem

With a certain loss of quality in the results, which will be sub-optimal, the problem can be addressed by splitting it into 3 parts, and applying a different resolution technique on each part.

Parallel machine problem
Broccoli/Cauliflower overwrapped

LOT Sizing problema
Constraints

Single machine problem
Broccoli&Cauliflower mix tray

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Solve the problem

Lot Sizing problem: A smaller and more affordable problem -> Solver can be used.

$$F.O.\ max\ z = \{\sum_{i=1}^{n}\left(\mathrm{q}_i * I_i - (D_i - k_i) * PTY_i - \left(\sum_{t=1}^{q}(xb_{t,i} * BC_t\right) + \sum_{tt=1}^{qq}(xc_{tt,i} * CC_{tt})\right) - \left(\sum_{t=1}^{q} xb_{t,i} + \sum_{tt=1}^{qq} xc_{tt,i} - k_i\right) * CT_i)\} \quad (1')$$

- $Qb_t \geq \sum_{i=1}^{n} xb_{t,i} \quad \forall t$ (2)
- $Qc_{tt} \geq \sum_{i=1}^{n} xb_{tt,i} \quad \forall tt$ (3)
- $D_i \geq k_i \ \forall i$ (4)
- $k_i = \sum_{t=1}^{q}\sum_{t'}^{q}\left(xb_{t,i} * QBM_{t,t'} * BR_{t',i}\right) + \sum_{tt=1}^{qq}\sum_{tt'}^{q}\left(xc_{tt,i} * QCM_{tt,tt'} * CR_{tt',i}\right) \ \forall i$ (5')

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Solve the problem

- Scheduling problems are usually complex-> NP-hard problems.
- Model can be used to understand the problem and introduce alternatives.
- To solve the problem the most convenient option is use an heuristic or a metaheuristic algorithm (under-optimal).
- Two options (heuristic / Metaheuristic) are introduced now.

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Solve the problem

- Consider 2 scheduling problems:
  - Minimizing Total Tardiness in Unrelated Parallel Machine Scheduling problem.
  - Minimizing Total Tardiness in single Machine Scheduling problem.
- In this approach I use the same algorithm for both.

Parallel machine problem
Broccoli/Cauliflower overwrapped

Single machine problem
Broccoli&Cauliflower mix tray

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Solve the problem

Any of 2 scheduling problems can be model as:

$$F.O.\ min\ z = \{\sum_{i=1}^{n} u_i * PY_i\} \quad (1)$$

- $\sum_{j=1}^{m} wc_{j,i} = 1 \quad \forall i$ (6)
- $\sum_{j=1}^{m} wc_{j,i} * WC_{j,i} = 1 \quad \forall i$ (7)
- $w_{j,i} + \sum_{r=1/r\neq j}^{m} w_{r,l} + y_{i,l} \leq 2 \quad \forall i\ \forall l\ \forall j$ (8)
- $c_l - c_i + \mathrm{M} * (3 - \mathrm{y}_{i,l} - w_{j,i} - w_{j,l}) \geq P_{j,l} * \mathrm{k}_l \quad \forall i, \forall l, \forall j$ (9)
- $c_i - c_l + \mathrm{M} * (2 + \mathrm{y}_{i,l} - w_{j,i} - w_{j,l}) \geq P_{j,i} * \mathrm{k}_i \quad \forall i, \forall l, \forall j$ (10)
- $c_i + \mathrm{M} * (1 - \mathrm{wc}_{j,i}) \geq P_{j,i} * \mathrm{k}_i \quad \forall i\ \forall j$ (11)
- $(L_i - W_i - c_i) \leq u_i$ (12)

This is the second part of the original model.

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Solve the problem

Minimizing Total Tardiness in Unrelated Parallel (single) Machine Scheduling problem.

A simple heuristic is: SH

1. Create a list L of unprocessed orders.
2. Create list L with $\left(L_i - W_i - c_{i,j}\right) = u_{i,j}$ for each order i in L for each machine j.
3. Order in U list all $u_{i,j}$ from least to greatest.
4. Select the first element of list U and assign order i to machine j.
5. Remove from the list L order i. Go to step 2.

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Solve the problem

Minimizing Total Tardiness in Unrelated Parallel (single) Machine Scheduling problem.

A metaheuristic:

GRASP: Greedy Randomized Adaptative Search Procedure

S_{best}<-ConstructRandomSolution()

While (Not StopCondition())

$S_{candidate}$<-GreedyRandomizedConstruction(alfa)

$S_{candidate}$<-LocalSearch($S_{candidate}$)

If ($U(S_{candidate}<-) < U(S_{best})$) S_{best} <-$S_{candidate}$

Return (S_{best})

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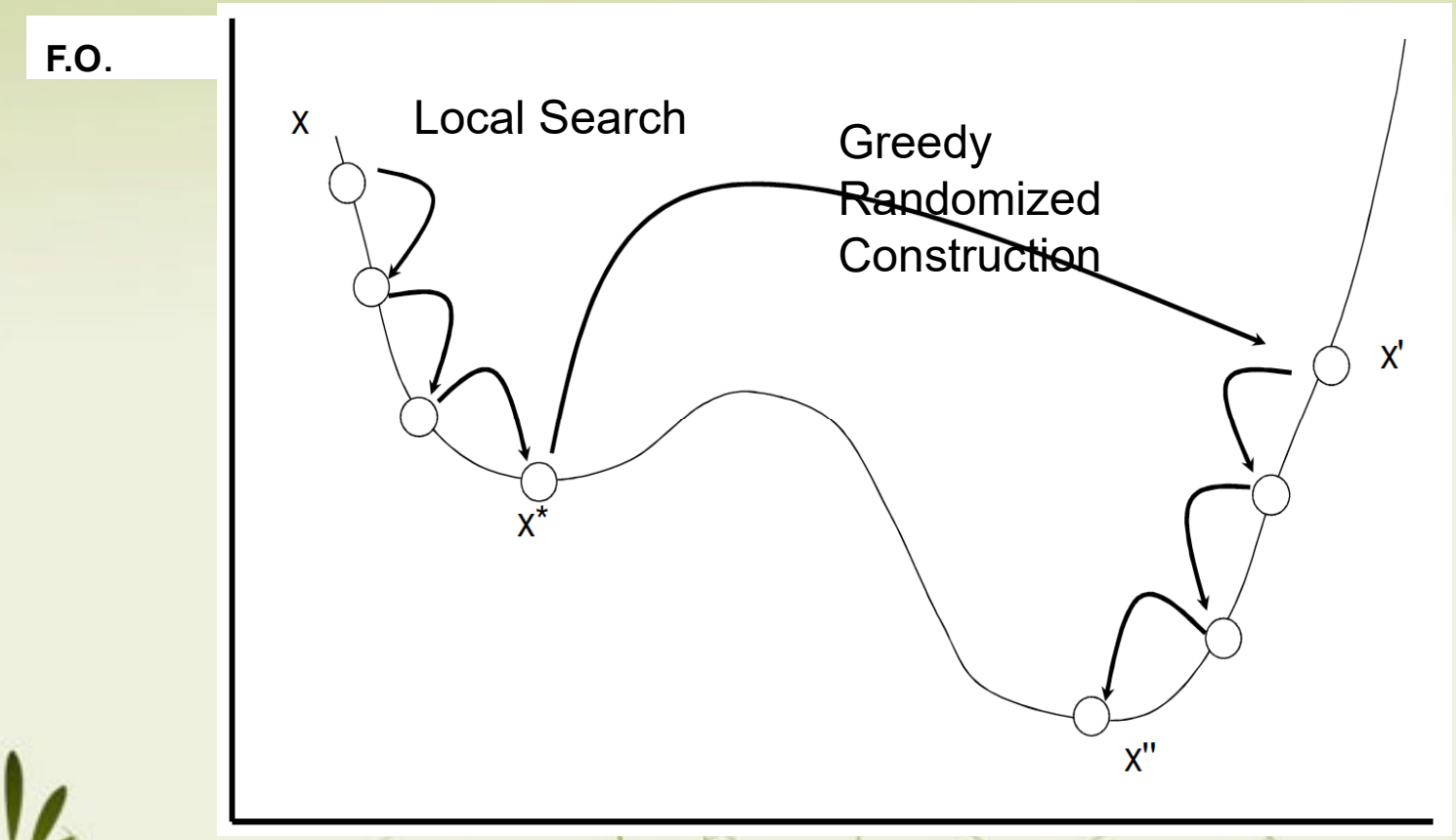
Solve the problem

GRASP: Greedy Randomized Adaptative Search Procedure



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Solve the problem

GRASP: Greedy Randomized Adaptative Search Procedure

– GreedyRandomizedConstruction(alfa) -> Re-using a modified version of SH.

1. Create a list L of unprocessed orders.
2. Create list L with $\left(L_i - W_i - c_{i,j}\right) = u_{i,j}$ for each order i in L for each machine j.
3. Order in U list all $u_{i,j}$ from least to greatest.
4. Select the one element of list U among top "alfa" and assign order i to machine j.
5. Remove from the list L order i. Go to step 2.

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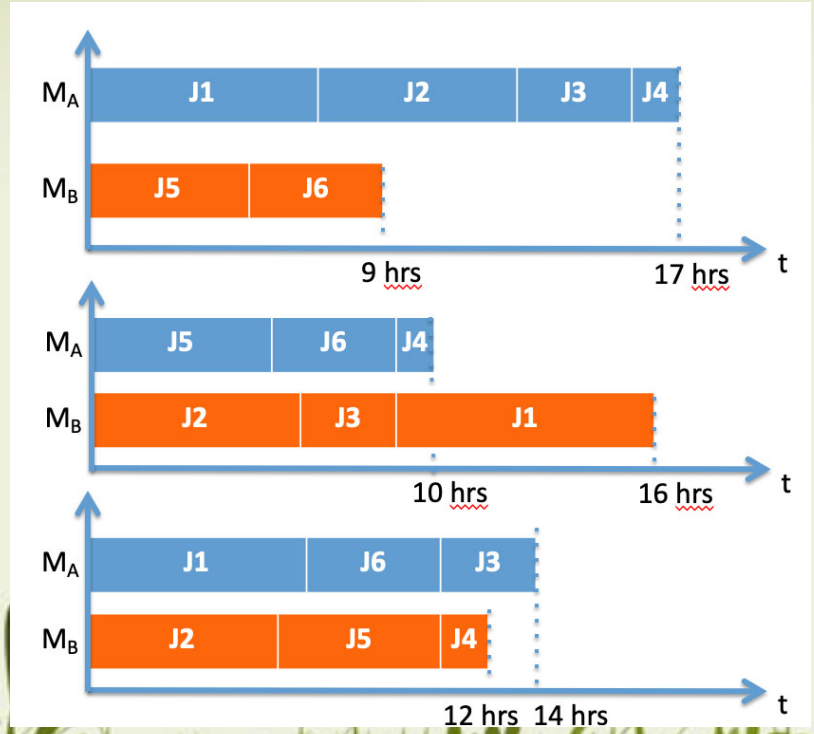
Solve the problem

GRASP: Greedy Randomized Adaptative Search Procedure



M_A	1	2	3	4
M_B	5	6	-	-

M_A	5	6	4	-
M_B	2	3	1	-

M_A	1	6	3	-
M_B	2	5	4	-

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Solve the problem

GRASP: Greedy Randomized Adaptative Search

M_A	5	6	4	-
M_B	2	3	1	-

M_A	4	6	5	-
M_B	2	3	1	-

LocalSearch($S_{candidate}$)

1. Random swap in the same machine.
2. Random swap between machines.

M_A	5	2	4	-
M_B	6	3	1	-

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Next Steps

- Implement some numerical cases and analyse preliminary results.
- Implement algorithms and link them all for a global solution.
- Define new models considering:
  - Delivery dates in distribution.
  - non-compliant product feedback on the same period.
  - Introduce uncertainty using fuzzy sets.

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Thank you

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